

PRAAAS

JEE 2026

Mathematics

Basic Maths

Lecture - 17

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Topics *to be covered*



- A** Exponential Inequalities
- B** Characteristic & Mantissa
- C** Problem Practice



Homework Discussion

Bumper Practice Problems



$$1. \sqrt[3]{\frac{x-2}{x-1}} < \sqrt[3]{\frac{1}{x-1}}$$

$$2. \sqrt{3x-2} < \sqrt{x+4}$$

$$3. \sqrt{4x-3} < \sqrt{2x+5}$$

$$4. \sqrt[3]{(3x-5)} < (x-1)$$

$$5. \sqrt[3]{3x-2} < x$$

$$6. \sqrt{(x+14)} < (x+2)$$

$$7. \sqrt{2x-2} < x-1$$

$$8. \sqrt{-x^2+4x-3} > (6-2x)$$

$$9. \sqrt{x-2} + \sqrt{x-1} > 2$$

$$10. 7\sqrt{x} + 8\sqrt{-x} + \frac{15}{x^3} = 98$$

$x > 0$ (under $\sqrt{x}$)
 $-x > 0 \Rightarrow x \leq 0$ (under $\sqrt{-x}$)
 $x \neq 0$ (under $\frac{15}{x^3}$)
 $x \in \emptyset$ (next to the equation)

$$11. \sqrt{2x-4} - \sqrt{x+5} = 1$$

$$12. \sqrt{x-1} + \sqrt{2x+6} = 6$$

1. $x \in (1, 3)$

2. $x \in \left[\frac{2}{3}, 6\right)$

3. $x \in \left[\frac{3}{4}, 1\right)$

4. $x \in (-1, \infty) - \{2\}$

5. $x \in (-2, 1) \cup (1, \infty)$

6. $x \in (2, \infty)$

7. $x \in (3, \infty)$

8. $x \in \left(\frac{13}{5}, 3\right)$

9. $x \in \left(\frac{41}{16}, \infty\right)$



$$\textcircled{9} \sqrt{x-2} + \sqrt{x-1} > 2$$

S.B.S

$$x-2 + x-1 + 2\sqrt{(x-1)(x-2)} > 4$$

$$2x-3 + 2\sqrt{(x-1)(x-2)} > 4$$

$$2\sqrt{(x-1)(x-2)} > 7-2x$$

case (i) if $7-2x \geq 0 \Rightarrow 2x \leq 7 \Rightarrow x \leq 7/2$

S.B.S $4(x^2 - 3x + 2) > 49 + 4x^2 - 28x$

$$-12x + 8 > 49 - 28x$$

$$16x > 41$$

$$x > \frac{41}{16}$$

$$x-2 \geq 0 \text{ \& \& } x-1 \geq 0$$

$$x \geq 2 \text{ \& } x \geq 1$$

$$x \geq 2 \rightarrow \textcircled{A}$$

$$\frac{41}{16} < \frac{7}{2}$$

$$41 < 56$$

case (ii) if $7-2x < 0 \Rightarrow x > \frac{7}{2}$

$$2\sqrt{(x-1)(x-2)} > 7-2x$$

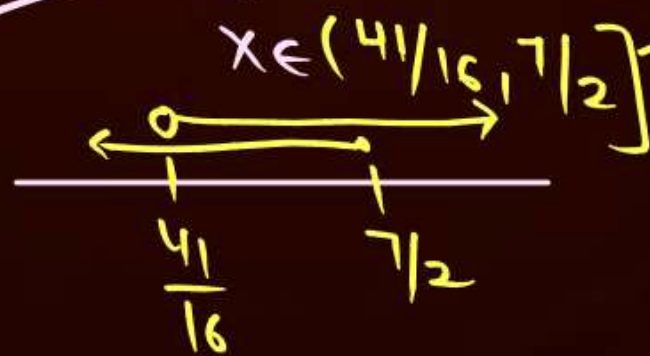
(always true)

$$x \in (7/2, \infty)$$

UNION

$$x \in (\frac{41}{16}, \infty) \rightarrow \textcircled{B}$$

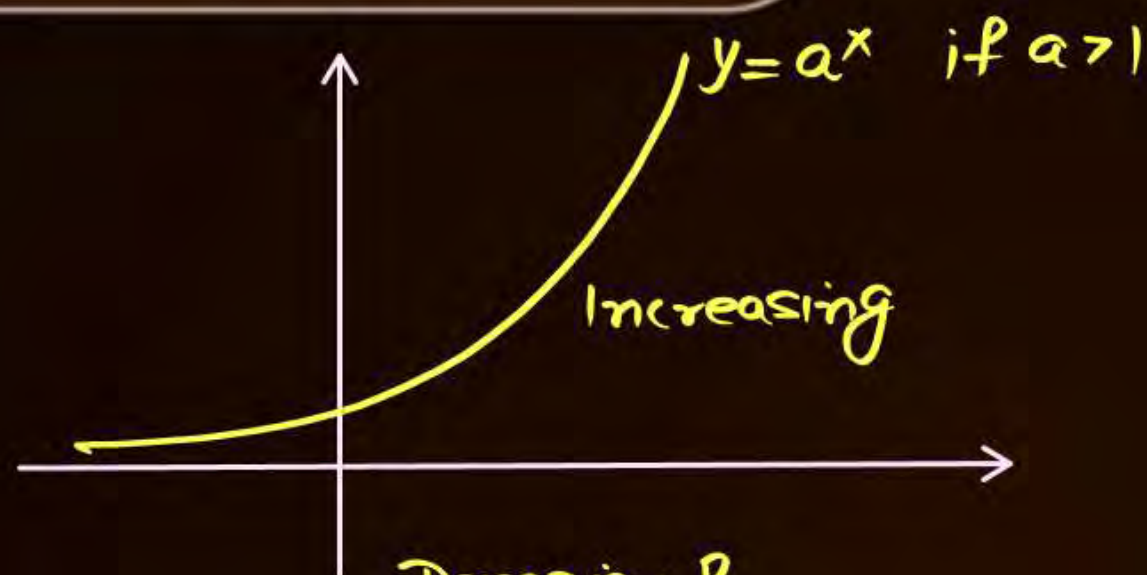
Ans: $A \cap B \Rightarrow (\frac{41}{16}, \infty)$



**Aao Machaay Dhamaal
Deh Swaal pe Deh Swaal**



Exponential Inequalities



Domain: $\mathbb{R}$
Range: $\mathbb{R}^+$

Ex: $3^{x-2} > 3^{2x-3}$ find range of x

$$x-2 > 2x-3$$

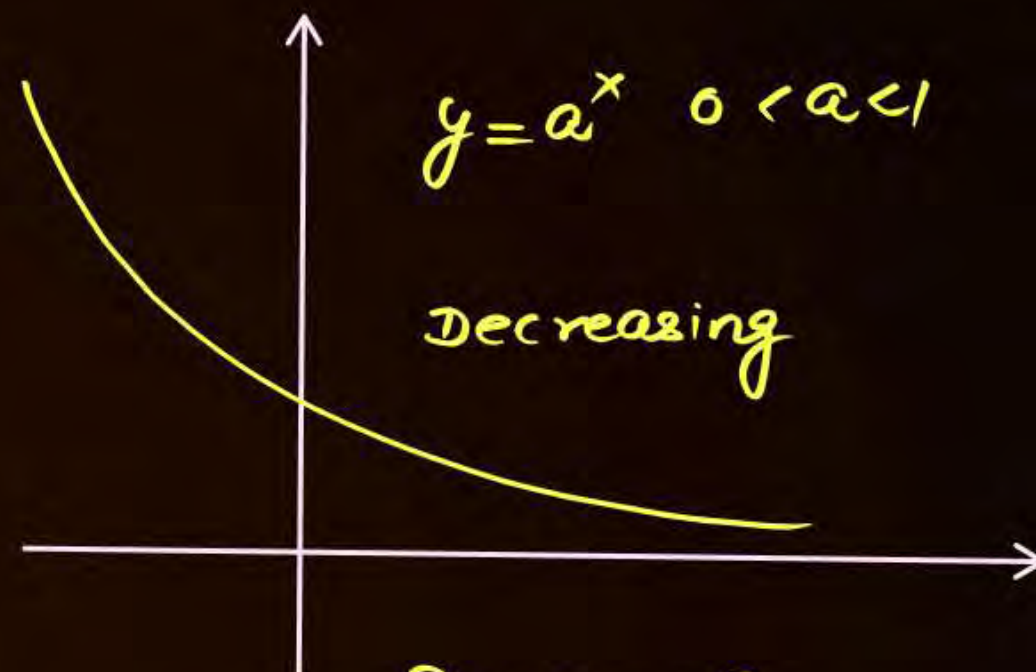
$$x < 1 \Rightarrow x \in (-\infty, 1)$$

Ex: $\left(\frac{1}{2}\right)^{x^2-2x} > \left(\frac{1}{2}\right)^{4x-8}$ find range of x .

$$x^2 - 2x < 4x - 8 \Rightarrow x^2 - 6x + 8 < 0$$

$$y = a^x \quad 0 < a < 1$$

Decreasing



Domain: $\mathbb{R}$
Range: $\mathbb{R}^+$

$$(x-2)(x-4) < 0$$
$$x \in (2, 4)$$

(a) $(3)^{\log_2(x^2-3x+2)} > 3'$

$$\log_2(x^2-3x+2) > 1 \quad \& \quad x^2-3x+2 > 0$$

(NO Need)

$$x^2-3x+2 > 2$$

$$x(x-3) > 0$$

↓

$$x \in (-\infty, 0) \cup (3, \infty)$$

(b) $(1/3)^{(x^2+2x)} > (1/9)^{16-x}$

$$\left(\frac{1}{3}\right)^{x^2+2x} > \left(\frac{1}{3}\right)^{32-2x}$$

$$x^2+2x < 32-2x$$

$$x^2+4x-32 < 0$$

$$x^2+8x-4x-32 < 0$$

$$(x+8)(x-4) < 0$$

$$x \in (-8, 4) \quad \underline{\text{Ans}}$$

M② Taking log to base $1/3$ on both sides

$$\log_{\frac{1}{3}}(1/3)^{x^2+2x} < \log_{\frac{1}{3}}(1/3)^{32-2x}$$

$$(x^2+2x) < 32-2x$$

$$x^2+4x-32 < 0$$

$$\rightarrow x \in (-8, 4)$$

QUESTION



(a) $2^x > 5$

Taking log to base 2 on both sides.

$$\log_2 2^x > \log_2 5$$

$$x \log_2 2 > \log_2 5$$

$$x > \log_2 5$$

(b) $(1/2)^{\log_3(x^2-2x-3)} > 1$

$$\left(\frac{1}{2}\right)^{\log_3(x^2-2x-3)} > \left(\frac{1}{2}\right)^0$$

$$\log_3(x^2-2x-3) < 0 \quad \& \quad x^2-2x-3 > 0$$

$$x^2-2x-3 < 3^0$$

$$x^2-2x-3 < 1$$

$$x^2-2x-4 < 0$$

$$x = \frac{2 \pm \sqrt{20}}{2} = 1 \pm \sqrt{5}$$

$$(x - (1 + \sqrt{5}))(x - (1 - \sqrt{5})) < 0$$

$$x \in (1 - \sqrt{5}, 1 + \sqrt{5})$$

$$\downarrow$$

$$(x-3)(x+1) > 0$$

$$x \in (-\infty, -1) \cup (3, \infty)$$

$$\cap$$

$$x \in (1 - \sqrt{5}, -1) \cup (3, \sqrt{5} + 1)$$

QUESTION

Tahoi



Find the number of integral values of x satisfying the inequality

$$\left(\frac{3}{4}\right)^{6x+10-x^2} < \frac{27}{64} = \left(\frac{3}{4}\right)^3$$

QUESTION [JEE Mains 2020 (8 Jan)]

Tah02



Let S be the set of all real roots of the equation,
 $3^x(3^x - 1) + 2 = |3^x - 1| + |3^x - 2|$. Then S :

- A** contains exactly two elements.
- B** is an empty set.
- C** is a singleton.
- D** contains at least four elements.

Ans. C

Tan03

The number of real roots of the equation $5 + |2^x - 1| = 2^x(2^x - 2)$ is

A 2

B 1

C 3

D 4

Case ① $2^x - 1 \geq 0$
 $2^x \geq 1 = 2^0$
 $x \geq 0$

Case ② $2^x - 1 < 0 \rightarrow 2^x < 1 = 2^0 \Rightarrow x < 0$

Tahol

If x is a solution of the equation, $\sqrt{2x+1} - \sqrt{2x-1} = 1$, $\left(x \geq \frac{1}{2}\right)$, then $\sqrt{4x^2 - 1}$ is equal to:

- A** $\frac{3}{4}$
- B** $\frac{1}{2}$
- C** 2
- D** $2\sqrt{2}$

QUESTION [JEE Mains 2022 (30 June)]



Tan 05

Let $S_1 = \left\{x \in \mathbb{R} - \{1, 2\} : \frac{(x+2)(x^2+3x+5)}{-2+3x-x^2} \geq 0\right\}$ and $S_2 = \{x \in \mathbb{R} : 3^{2x} - 3^{x+1} - 3^{x+2} + 27 \leq 0\}$.

Then, $S_1 \cup S_2$ is equal to :

$$3^{2x} - 3^x \cdot 3 - 9 \cdot 3^x + 27 \leq 0$$

$$\text{let } 3^x = t$$

A $(-\infty, -2] \cup (1, 2)$

B $(-\infty, -2] \cup [1, 2]$

C $(-2, 1] \cup [2, \infty)$

D $(-\infty, 2]$

Ans. B

QUESTION [JEE Mains 2022 (25 June)]

Tah06



Let $A = \{x \in \mathbb{R} : |x + 1| < 2\}$ and $B = \{x \in \mathbb{R} : |x - 1| \geq 2\}$.
Then which one of the following statements is NOT true?

- A** $A - B = (-1, 1)$
- B** $B - A = \mathbb{R} - (-3, 1)$
- C** $A \cap B = (-3, -1]$
- D** $A \cup B = \mathbb{R} - [1, 3)$

Ans. B

QUESTION [JEE Mains 2021 (18 March)]



The value of $3 + \frac{1}{4 + \frac{1}{3 + \frac{1}{4 + \frac{1}{3 + \dots \infty}}}}$ is equal to

- ☒ A $1.5 + \sqrt{3}$
- ☐ B $2 + \sqrt{3}$
- ☐ C $3 + 2\sqrt{3}$
- ☐ D $4 + \sqrt{3}$

$$x = 3 + \frac{1}{4 + \frac{1}{3 + \frac{1}{4 + \frac{1}{3 + \dots \infty}}}}$$

$$x = 3 + \frac{1}{4 + \frac{1}{x}}$$

$$x = 3 + \frac{x}{4x+1} = \frac{12x+3+x}{4x+1}$$

$$4x^2+x=13x+3$$

$$4x^2-12x-3=0$$

$$x = \frac{12 \pm \sqrt{144+48}}{8}$$

$$x = \frac{12 \pm 4\sqrt{9+3}}{8} = \frac{3 \pm 2\sqrt{3}}{2}$$

$$x = 1.5 + \sqrt{3}$$

Ans. A

QUESTION [JEE Mains 2021 (17 March)]



Tah 07

The value of $4 + \frac{1}{5 + \frac{1}{4 + \frac{1}{5 + \frac{1}{4 + \dots \infty}}}}$ is :

- A** $2 + \frac{2}{5}\sqrt{30}$
- B** $2 + \frac{4}{\sqrt{5}}\sqrt{30}$
- C** $5 + \frac{2}{5}\sqrt{30}$
- D** $4 + \frac{4}{\sqrt{5}}\sqrt{30}$

Ans. A

The product of the roots of the equation $9x^2 - 18|x| + 5 = 0$ is :

- A** $\frac{5}{9}$
- B** $\frac{5}{27}$
- C** $\frac{25}{81}$
- D** $\frac{25}{9}$

The sum of all the real values of x satisfying the equation $2^{(x-1)(x^2+5x-50)} = 1$ is

- A** 16
- B** 14
- C** -4
- D** -5

Antilog



$$\text{antilog}_{10}(\log_{10} 20) = 20$$

$$\text{antilog}_2(\log_2 6) = 6$$

$$\text{antilog}_a(\log_a b) = b$$

$$\text{let } \log_a b = x$$

$$b = a^x$$

$$\text{antilog}_a x = a^x$$

$$\text{Ex: } \text{antilog}_2 3 = 2^3 = 8$$

$$\text{Ex: } \text{antilog}_{10} 100 = 10^{100}$$



Antilogarithm

$$(a) \quad \text{antilog}_{100} \left(-\frac{1}{2} \right) = 100^{-\frac{1}{2}} = \frac{1}{\sqrt{100}} = \frac{1}{10} = 0.1$$

$$(b) \quad \text{antilog}_8 \left(\frac{2}{3} \right) = 8^{2/3} = \left(8^{1/3} \right)^2 = 2^2 = 4$$



Characteristic & Mantissa



logarithm of any no. to a given base always has two parts an integral part called **characteristic** and a fractional part called **mantissa**

Ex: $\log_2 16 = 4.0$

Integral part $\rightarrow$ fractional part

Characteristic = 4 $\rightarrow$ Mantissa = 0

Characteristic $\in$ Integer.
Mantissa $\in [0, 1)$

Ex: find characteristic of $\log_2 17$

clearly: $2^4 < 17 < 2^5 \rightarrow 4 < \log_2 17 < 5 \rightarrow \log_2 17 = 4 \cdot \text{something}$
characteristic = 4

QUESTION



Find the number of positive integers which have the characteristic 3 if the base of the logarithm is 5.

let $\log_5 N = 3 \cdot \text{something}$.

$$3 \leq \log_5 N < 4$$

$$5^3 \leq N < 5^4$$

$$125 \leq N < 625.$$

No. of possible integral values of $N = 500$ Ans.

$$1 \leq N < 10$$

$$N = 1, 2, 3, 4, 5, 6, 7, 8, 9$$

No. of values of $N = 9$

Ex: find No: of possible +ve integers for which $\log$ to base 7 has characteristic 2.

$$\log_7 N = 2 \cdot \text{something}$$

$$2 \leq \log_7 N < 3$$

$$49 \leq N < 343$$

$$\begin{aligned} \text{No: of Integral values of } N &= 343 - 49 \\ &= 294 \text{ Ans.} \end{aligned}$$

Ex: find characteristic of $\log_4 500$

$$4^4 < 500 < 4^5$$

$$4 < \log_4 500 < 5$$

$$\log_4 500 = 4 \cdot \text{something} \quad \text{characteristic} = 4$$

$$* a \leq x < b \quad x, a, b \in \mathbb{I}$$

No: of possible values of $x = b - a$

$$* a < x \leq b, \quad x, a, b \in \mathbb{I}$$

No: of possible values of $x = b - a$

$$* a \leq x \leq b, \quad x, a, b \in \mathbb{I}$$

No: of possible values of $x = b - a + 1$

$$* a < x < b, \quad a, b, x \in \mathbb{I}$$

No: of values of $x = b - a - 1$

$b - a$ se eak end point answer mai shamil hota hai

$\log_{10} N$ KIKHAANI

$$f \in [0, 1)$$

Ex: $\log_2 1 = 0$, $\log_{10} 1 = 0$
 Ex: $\log_2 0$ is not defined in reals

If $N > 1$

$N = \overleftarrow{1.63}, \overleftarrow{9.85} \quad * \quad 1 \leq N < 10 \Rightarrow 0 \leq \log_{10} N < 1 \Rightarrow \log_{10} N = 0 + f$

$N = \overleftarrow{95.62}, \overleftarrow{88.55} \quad * \quad 10 \leq N < 100 \Rightarrow 1 \leq \log_{10} N < 2 \Rightarrow \log_{10} N = 1 + f$

$N = \overleftarrow{110.23}, \overleftarrow{999.25} \quad * \quad 100 \leq N < 1000 \Rightarrow 2 \leq \log_{10} N < 3 \Rightarrow \log_{10} N = 2 + f$

characteristic

0

1

2

If $N > 1$, $\log_{10} N$ has characteristic = $\left(\text{No. of significant digits to left of decimal in } N \right) - 1$

Kallu: $N = \overleftarrow{2567.53}$

$\log_{10} N$ characteristic = $4 - 1 = 3$

Lallu ~~$N = \overleftarrow{0002567.53}$~~

~~$\log_{10} N = 7 - 1 = 6$~~

$N = 0.\overrightarrow{00(5)0708}$

$\log_{10} N$ characteristic = $-(2 + 1) = -3$

~~$N = \overrightarrow{0.0050708}$~~

~~$\log_{10} N = -(4 + 1) = -5$~~

$\log_{10} N$ क़ाहाANI

$$f \in [0, 1)$$

Ex: $\log_2 1 = 0$, $\log_{10} 1 = 0$
 Ex: $\log_2 0$ is not defined in reals.

if $0 < N < 1$

$$N = \overset{\rightarrow}{0.689}, \overset{\rightarrow}{0.976} * \frac{1}{10} \leq N < 1 \Rightarrow -1 \leq \log_{10} N < 0 \Rightarrow \log_{10} N = -1 + f$$

$$N = \overset{\rightarrow}{0.078}, \overset{\rightarrow}{0.09705} * \frac{1}{100} \leq N < \frac{1}{10} \Rightarrow -2 \leq \log_{10} N < -1 \Rightarrow \log_{10} N = -2 + f$$

$$N = \overset{\rightarrow}{.0078}, \overset{\rightarrow}{0.00965} * \frac{1}{1000} \leq N < \frac{1}{100} \Rightarrow -3 \leq \log_{10} N < -2 \Rightarrow \log_{10} N = -3 + f$$

characteristic

-1

-2

-3

if $0 < N < 1$, $\log_{10} N$ has characteristic = - (No. of 0's immediately to right of decimal in N before a +1 significant digit starts)

Common logarithm of N is logarithm of N to base 10 . i.e. $\log_{10} N$

Natural logarithm of N is logarithm of N to base e . i.e., $\log_e N = \ln N$

Note :

Logarithm of an positive real number N to base $a > 0, a \neq 1$ always has two parts

- (1) An integral part called as characteristic.
- (2) A fractional part called as mantissa fraction part always lies in $[0, 1)$

QUESTION



Using $\log 2 = 0.3010$ and $\log 3 = 0.4771$
Find the number of digits

(i) $(2)^{200}$

$$x = 2^{200}$$

$$\log_{10} x = \log_{10} 2^{200} = 200 \log_{10} 2$$

$$\log_{10} x = 200 \times 0.3010$$

$$\log_{10} x = 60.2 = \overbrace{60}^{\text{characteristic}} + 0.2$$

$$\left(\begin{array}{l} \text{No. of digits} \\ \text{in } x \text{ to left} \\ \text{of decimal} \end{array} - 1 \right) = \text{characteristic} = 60$$

$$\downarrow$$

$$\text{No. of digits in } x = 61$$

(ii) $3^{12} \times 2^8 = x$

$$x = 3^{12} \times 2^8$$

$$\log_{10} x = \log_{10} (3^{12} \times 2^8) = 12 \log_{10} 3 + 8 \log_{10} 2$$

$$\log_{10} x = 12 \times 0.4771 + 8 \times 0.3010$$

$$\log_{10} x = 8.1332$$

$$\text{No. of digit in } x = 9$$

136048896

QUESTION



$$\log_{10} 7 = 0.8451$$

Find the number of zeroes after decimal before a significant figures start in

(i) $\left(\frac{9}{8}\right)^{-100}$

(ii) $(0.35)^{12}$

(iii) $\frac{1}{2^{40}}$

$$x = \left(\frac{9}{8}\right)^{-100}$$

$$0.000007669159 - -$$

$$\log_{10} x = -100(\log 9 - \log 8)$$

$$= -100(2 \log 3 - 3 \log 2)$$

$$= -100(2 \times 0.4771 - 3 \times 0.3010)$$

$$\log_{10} x = -5.12 = -5 - 0.12 = -5 - 1 + 1 - 0.12 = -6 + 0.88$$

$$\text{Characteristic} = -6$$

No. of zeros in x immediately after decimal = 5

$$-(n+1) = -6$$

$$n+1 = 6$$

$$n = 5$$

Mantissa is never negative It lies in $[0, 1)$

$$x = (0.35)^{12}$$

$$\log_{10} x = 12 \log_{10} (35/100)$$

$$= 12(\log_{10} 7 + \log_{10} 5 - 2)$$

$$= 12(0.8451 + \log_{10}(10/2) - 2)$$

$$= 12(0.8451 + 1 - \log_{10} 2 - 2)$$

$$= 12(0.8451 + 1 - 0.3010 - 2)$$

$$= -5.4708 = -5 - 1 + 1 - 0.4708$$

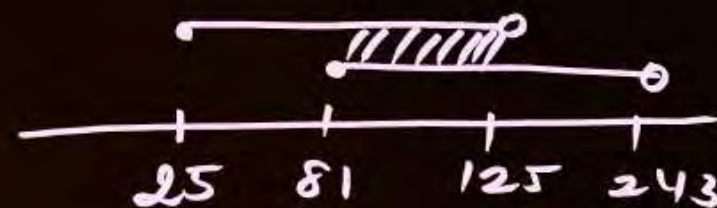
$$= -6 + 0.5292$$

No. of zeros in x
After decimal before s.d = 5 Ans

QUESTION



Let $\log_3 N = \alpha_1 + \beta_1$, $\log_5 N = \alpha_2 + \beta_2$ and $\log_7 N = \alpha_3 + \beta_3$ where $\alpha_1, \alpha_2, \alpha_3$ are integers and $\beta_1, \beta_2, \beta_3 \in [0, 1)$.



- (i) Find the number of integral values of N if $\alpha_1 = 4$ and $\alpha_2 = 2$
- (ii) Find the largest integral values of N if $\alpha_1 = 5$ and $\alpha_2 = 3$ and $\alpha_3 = 2$

Tah 10

$$\begin{array}{l} \log_3 N = \alpha_1 + \beta_1 \\ \log_5 N = \alpha_2 + \beta_2 \\ \log_7 N = \alpha_3 + \beta_3 \end{array} \quad \begin{array}{l} \text{--- (i) ---} \\ \text{--- [0, 1) ---} \end{array}$$

$\downarrow$ Mantissa
 $\downarrow$ Characteristic

$$\alpha_1 = 4, \alpha_2 = 2 \text{ (given)}$$

$$4 \leq \log_3 N = 4 + \beta_1 < 5 \rightarrow 3^4 \leq N < 3^5 \Rightarrow 81 \leq N < 243$$

$$2 \leq \log_5 N = 2 + \beta_2 < 3 \rightarrow 5^2 \leq N < 5^3 \Rightarrow 25 \leq N < 125$$



$$81 \leq N < 125$$

$$\text{No. of Integral values of } N = 125 - 81 = 44$$

If $\frac{a}{b} = \frac{c}{d}$ then

* componendo Dividendo $\frac{a+b}{a-b} = \frac{c+d}{c-d}$ OR $\frac{a-b}{a+b} = \frac{c-d}{c+d}$

* $\frac{a+b}{b} = \frac{c+d}{d}$ OR $\frac{a}{a+b} = \frac{c}{c+d}$

Ex: find α, β if $3\alpha + 2\beta = 13$

If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \dots$ then

* $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \dots = \frac{k_1 a_1 + k_2 b_1 + k_3 c_1 + \dots}{k_1 a_2 + k_2 b_2 + k_3 c_2 + \dots}$

* $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \dots = \frac{a_1 + b_1 + c_1 + \dots}{a_2 + b_2 + c_2 + \dots}$

* $\frac{\alpha-2}{1} = \frac{\beta-1}{2}$

$\frac{\alpha-2}{1} = \frac{\beta-1}{2} = \frac{3\alpha-6+2\beta-2}{3+4}$

$\frac{\alpha-2}{1} = \frac{\beta-1}{2} = \frac{13-8}{7} = 5/7$

$\alpha = 19/7, \beta = 1+2/7 = 16/7$



Sabse Important Baat



Sabhi Class Illustrations Retry Karnay hai...



Home Challenge-09



If the least integral value satisfying the equation

$\log_3 \sqrt{x^2 - 4x + 4} = 2^{\log_2(\log_3(|x|-2))}$ is α , then find the number of zeroes after decimal and before first significant digit in the number of $(\alpha)^{-4\alpha}$. [Ans. 9]



Today's KTK



No Selection $\xrightarrow[\text{Apnao IIT Jao}]{\text{TRISHUL}}$ **Selection with Good Rank**



1. The common value of x satisfying $\frac{x-1}{x+2} \geq 0$ and $\frac{2x-5}{x-2} \leq 1$ is
 (A) $(2, \infty]$ (B) $(2, 3]$ (C) $(-\infty, 3]$ (D) None of these
[Ans. B]
2. The set of values of x satisfying the inequality $2x - 7 < 4x - 2$ and $-5 \leq 2x + 6 < 4$ is given as
 (A) $\left[-\frac{11}{2}, \frac{-5}{2}\right]$ (B) $\left[-\frac{11}{2}, \frac{-5}{2}\right)$ (C) $\left(-\frac{5}{2}, -1\right)$ (D) None of these
[Ans. C]
3. The smallest integer k satisfying the inequality $\frac{x-5}{x^2+5x-14} > 0$ is
 (A) -7 (B) -6 (C) 6 (D) None of these
[Ans. B]
4. Number of integer values of x satisfying the inequality $\frac{x^2+6x-7}{|x+4|} < 0$ is
 (A) 6 (B) 7 (C) 8 (D) None of these
[Ans. A]

5. Set of real values of x satisfying $|x + 4| = 3x - 2$ is given as

- (A) $\left\{-\frac{1}{2}, 3\right\}$ (B) $\left\{-\frac{1}{2}\right\}$ (C) $\{3\}$ (D) None of these

[Ans. C]

6. The complete set of solution of $2|x + 1| + |x - 3| = 4$ is given by

- (A) $\left\{\frac{5}{3}\right\}$ (B) $\{-1\}$ (C) $\left\{-1, \frac{5}{3}\right\}$ (D) None of these

[Ans. B]

7. The inequality $\frac{|x-3|}{x^2-5x+6} \geq 2$ is given as

- (A) $\left[\frac{3}{2}, 2\right] \cup \left[2, \frac{5}{2}\right]$ (B) $\left[\frac{3}{2}, 2\right]$ (C) $\left[\frac{3}{2}, \frac{5}{2}\right]$ (D) $\left[\frac{3}{2}, 2\right)$ [Ans. D]

8. Values of x satisfying the equality $|x^2 + 8x + 7| = |x^2 + 4x + 4| + |4x + 3|$ for $x \in \mathbb{R}$ are

- (A) $(-2, \infty)$ (B) $\left(\frac{3}{4}, \infty\right)$ (C) $\{2\} \cup \left[-\frac{3}{4}, \infty\right)$ (D) $\left[-\frac{4}{3}, \infty\right)$ [Ans. C]

9. The equation $4^{\left(\frac{1}{x}-2\right)} = \frac{1}{2} \ln \sqrt{e}$ has the solution-

- (A) -1 (B) 1 (C) 2 (D) None [Ans. B]

10. Solution of the equation $2^{x+2} \cdot 27^{x(x-1)} = 9$ are given by-

- (A) $\log_2 (2/3), 1$ (B) $2, 1 - \log_2 3$ (C) $-2, 1 - \log_2 3$ (D) None of these [Ans. C]

11. If $x^{\left[\log_3 x^2 + (\log_3 x)^2 - 10\right]} = \frac{1}{x^2}$ then x is equal to

- (A) 9, 1/9 (B) 9, 1/81 (C) 1, 1/9 (D) 2, 2/9 [Ans. B]

12. Complete set of values of x satisfying the inequality $x - 3 < \sqrt{x^2 + 4x - 5}$ is :

- (A) $(-\infty, 5] \cup [1, \infty)$ (B) $(-5, 3]$ (C) $[3, 5)$ (D) $(-5, 3)$ [Ans. A]



Homework From Module

Prarambh (Topicwise) : Q1 to Q25

Prabal (JEE Main Level) : Q1 to Q33

Solution to Previous TAH

QUESTION



If $|x - 5| - |x - 7| = k$ then for

- A** $K = 2$ the equation has infinitely many solution
- B** $K = -2$ the equation has infinitely many solution
- C** $K \in (2, \infty)$ the equation has no solution
- D** $K \in (-2, 2)$ the equation has exactly one solution

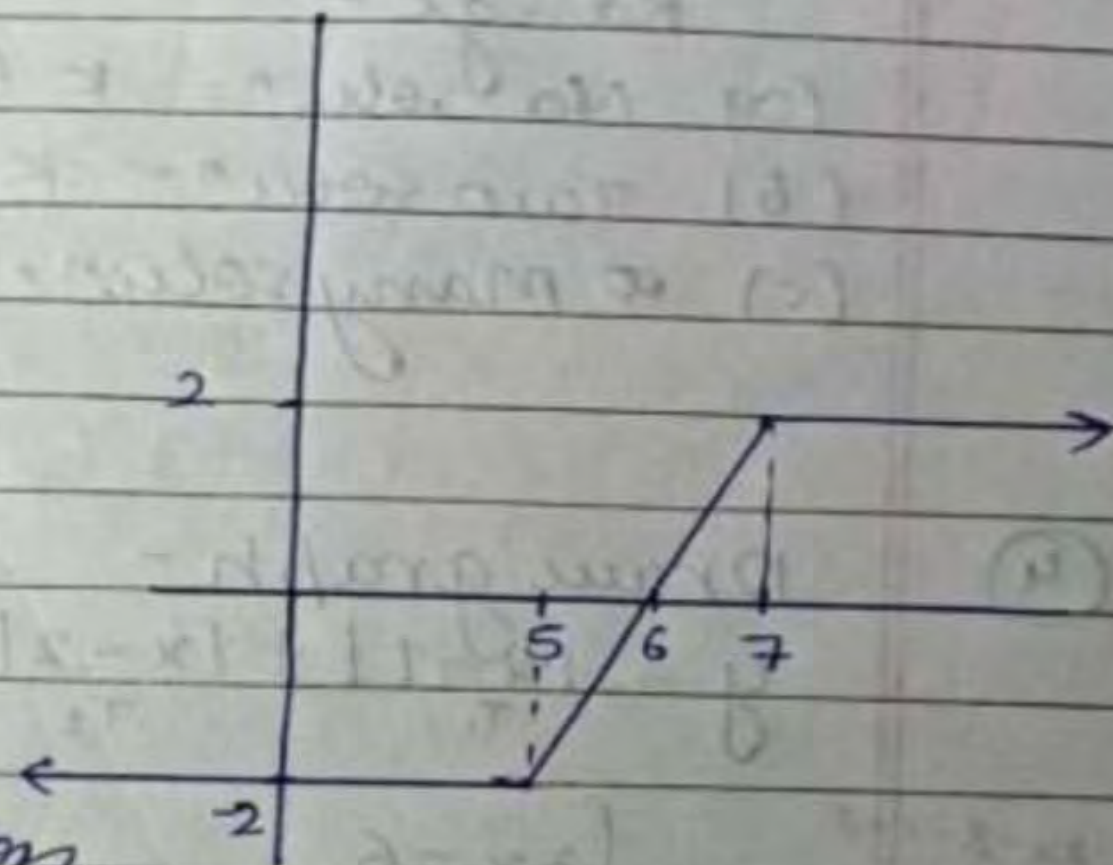
Tah 01

$$|x-5| - |x-7| = K$$

$$\begin{array}{c} T_1 \quad -ve \quad +ve \quad +ve \\ T_2 \quad -ve \quad 5 \quad -ve \quad 7 \quad +ve \end{array}$$

$$y = \begin{cases} -x+5+x-7 & x < 5 \\ x-5+x-7 & 5 \leq x < 7 \\ x-5-x+7 & x \geq 7 \end{cases}$$

$$y = \begin{cases} -2 & x \leq 5 \\ 2x-12 & 5 \leq x < 7 \\ 2 & x \geq 7 \end{cases}$$



- (A) $K=2$, ∞ many solutions
(B) $K=-2$, ∞ many solutions
(C) $K \in (2, \infty)$ eqn has no solution
(D) $K \in (-2, 2)$ the eqn has exactly one solution

all are correct

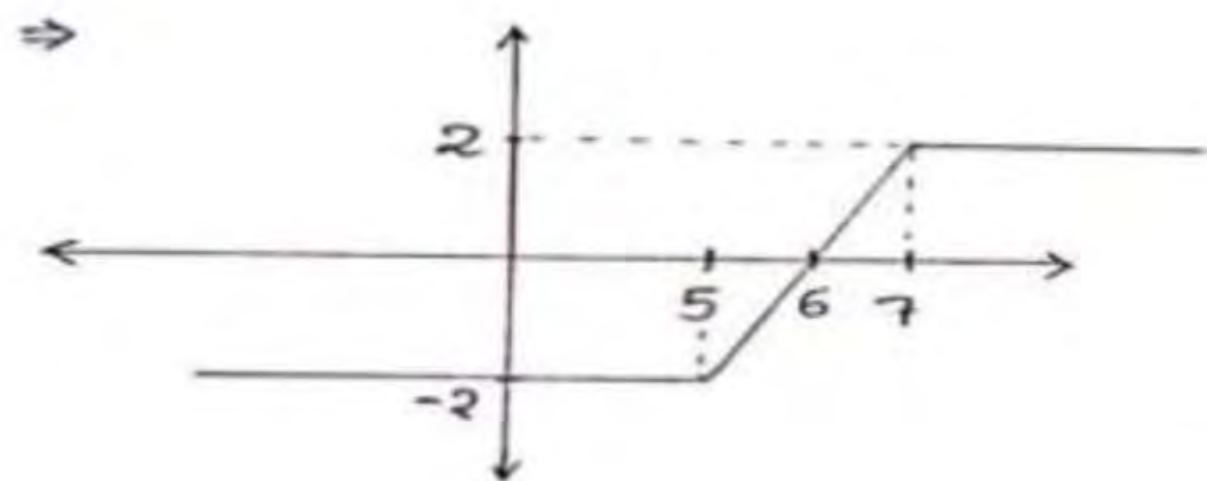
Tab-01. $|x-5| - |x-7| = k$ then for

- A) $k=2$ the eqⁿ has infinitely many solⁿ.
- B) $k=-2$ the eqⁿ has infinitely many solⁿ.
- C) $k \in (2, \infty)$ the eqⁿ has no solⁿ.
- D) $k \in (-2, 2)$ the eqⁿ has exactly one solⁿ.

$$\Rightarrow \begin{array}{c} T_1 \quad -ve \quad +ve \quad +ve \\ \hline \quad \quad 5 \quad \quad 7 \\ T_2 \quad -ve \quad -ve \quad +ve \end{array}$$

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$$\Rightarrow \left. \begin{array}{l} -x+5+x-7, \quad x \leq 5 \\ x-5+x-7, \quad 5 < x < 7 \\ x-5-x+7, \quad x \geq 7 \end{array} \right\} \Rightarrow \left. \begin{array}{l} -2, \quad x \leq 5 \\ 2x-12, \quad 5 < x < 7 \\ 2, \quad x \geq 7 \end{array} \right\}$$



(A)
(B)
(C)
(D)

All options correct.

Draw graph of $y = |x| + |x - 2|$ and answer the following.

Find the range of k for which the equation $|x| + |x - 2| = k$ has

- A. No Solution
- B. Infinitely Many Solutions
- C. Only two solutions

Tah-02. Draw graph of $y = |x| + |x-2|$ and answer the following. Find the range of K for which the eqⁿ:

$|x| + |x-2| = K$ has: $\Rightarrow$

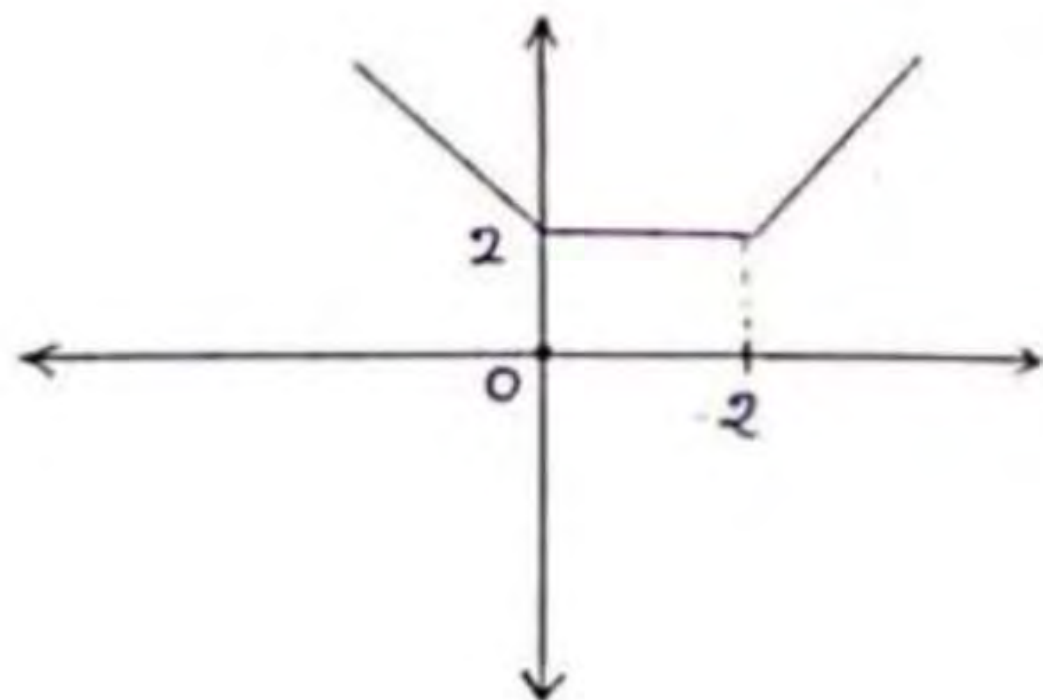
T_1	-ve	+ve	+ve
	$\xrightarrow{\hspace{1.5cm}}$		
	0	2	
T_2	-ve	-ve	+ve

(A) No solⁿ.

(B) Infinitely many solⁿ.

(C) Only two solⁿ.

$$\Rightarrow \left. \begin{aligned} -x - x + 2 &= -2x + 2, & x \leq 0 \\ x - x + 2 &= 2, & 0 < x < 2 \\ x + x - 2 &= 2x - 2, & x \geq 2 \end{aligned} \right\}$$



(A) $K \in (-\infty, 2)$.

(B) $K = 2$.

(C) $K \in (2, \infty)$.

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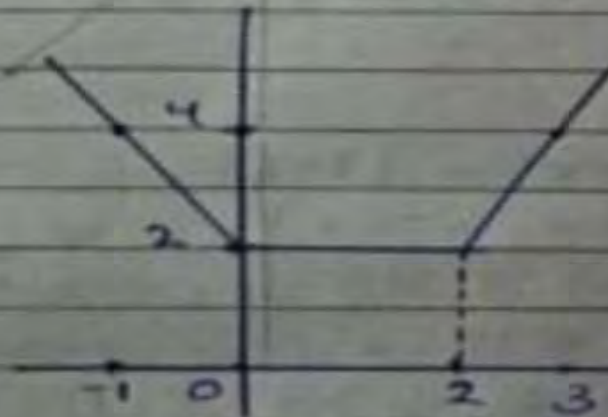
Ques 2

$$y = \overset{T_1}{|x|} + \overset{T_2}{|x-2|}$$

$$\begin{array}{l} T_1 \quad -ve \quad +ve \quad +ve \\ T_2 \quad -ve \quad 0 \quad -ve \quad +ve \end{array}$$

$$y = \begin{cases} -x-x+2 & x \leq 0 \\ x-x+2 & 0 < x < 2 \\ x+x-2 & x \geq 2 \end{cases}$$

$$y = \begin{cases} 2-2x & x \leq 0 \\ 2 & 0 < x < 2 \\ 2x-2 & x \geq 2 \end{cases}$$



Range of K :-

$$|x| + |x-2| = K$$

(A) No solution

$$K \in (-\infty, 2)$$

(B) Infinitely many solution

$$K = 2$$

(C) only one solution

$$K \in \phi$$

Draw the graph of $y = |x - 4| - |x + 3|$ and answer the following.

Find the range of k for which the equation $|x - 4| - |x + 3| = k$ has

- A. No Solution
- B. Infinitely Many Solutions
- C. Only two solutions

Tah-03.

Draw the graph of $y = |x-4| - |x+3|$ and answer the following. Find the range of k for which the eqⁿ $|x-4| - |x+3| = k$ has:

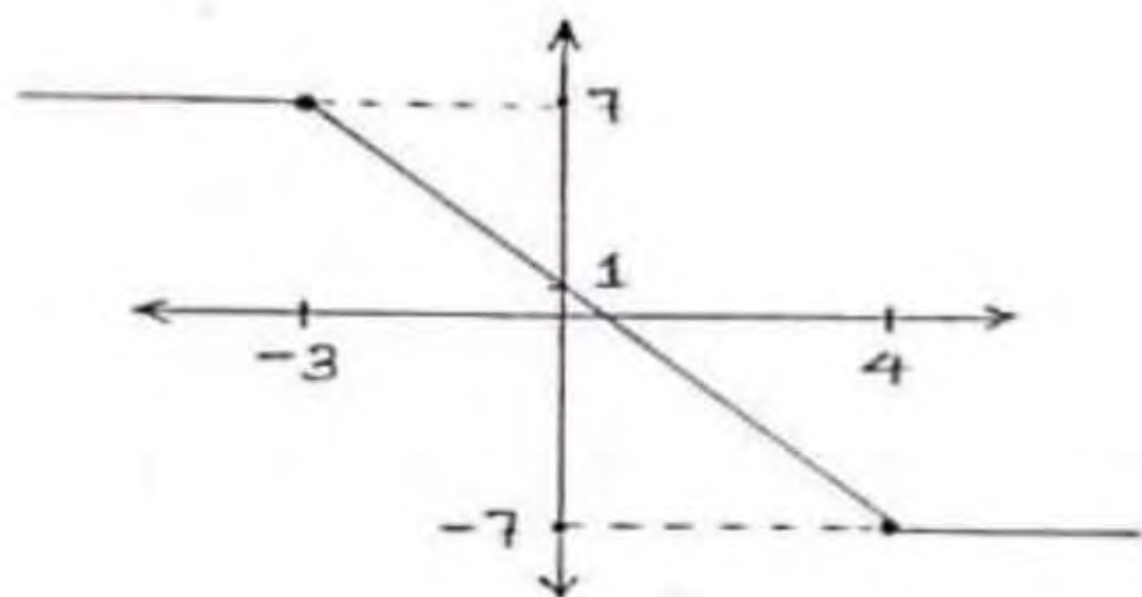
(A) NO solution.

(B) Infinitely many solⁿ.

(C) only two solⁿ.

$$\Rightarrow \begin{array}{c} T_1 \quad -ve \quad -ve \quad +ve \\ \hline \quad \quad -3 \quad \quad 4 \quad \quad \\ T_2 \quad -ve \quad +ve \quad +ve \end{array}$$

$$\Rightarrow \left. \begin{array}{l} -x+4+x+3, \quad x \leq -3. \\ -x+4-x-3, \quad -3 < x < 4. \\ x-4-x-3, \quad x \geq 4. \end{array} \right\}$$



$$\Rightarrow \left. \begin{array}{l} 7, \quad x \leq -3 \\ -2x+1, \quad -3 < x < 4 \\ -7, \quad x \geq 4 \end{array} \right\}$$

(A) $k \in (-\infty, -7) \cup (7, \infty)$.

(B) $k \in \{-7, 7\}$.

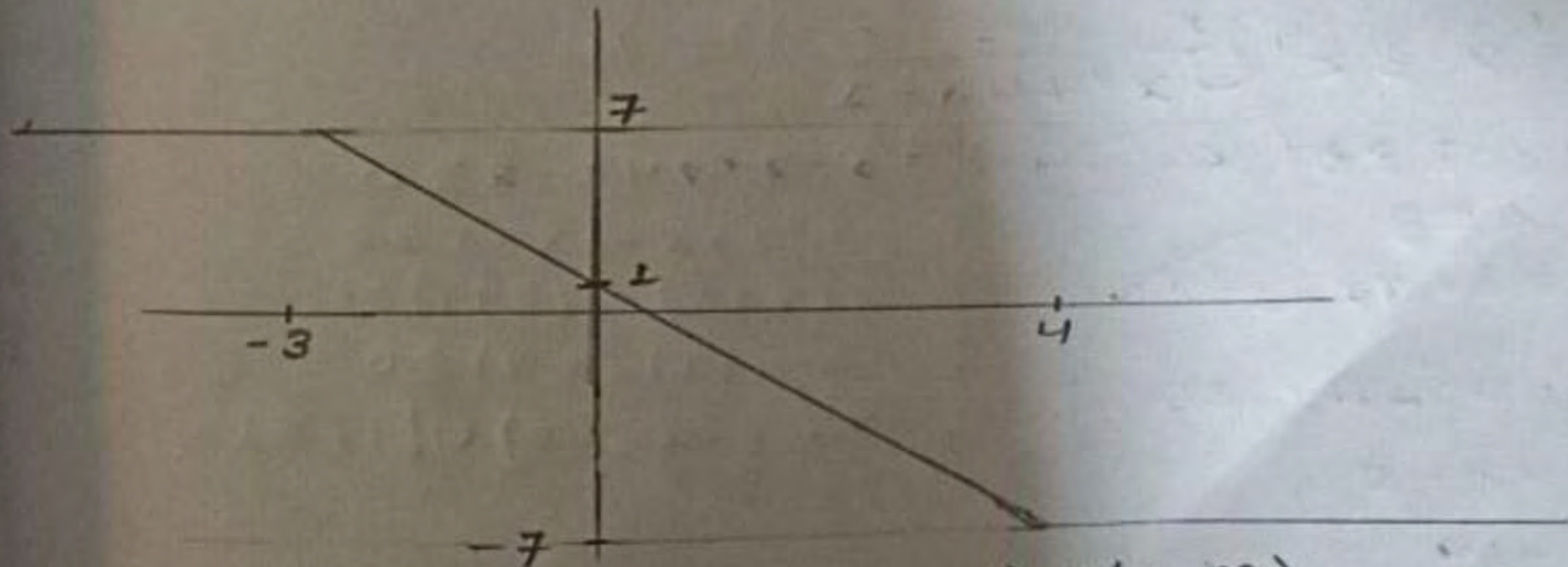
(C) $k \in \phi$.

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TAH03: $Y = |x^{T_1} - 4| - |x^{T_2} + 3|$

T_1	$-ve$	$-ve$	$+ve$
T_2	$-ve$	$+ve$	$+ve$
	-3	4	

$$Y = \begin{array}{lll} -x+4+x+3 & x \leq -3 & 7 \\ -x+4-x+3 & -3 < x < 4 & = -2x+1 \\ x-4-x-3 & x \geq 4 & -7 \end{array}$$



- A. No solution $\rightarrow K \in (-\infty, -7) \cup (7, \infty)$
 B. ∞ many soln $\rightarrow K \in (-7, 7)$
 C. only two soln $\rightarrow K \in (-7, 7)$

Consider the function $f(x) = |x - 1| - 2|x + 2| + |x + 3|$

Column-I

- (A) If $f(x) = k$ has no solution, then $k \in$
- (B) If $f(x) = k$ has one solution, then $k \in$
- (C) If $f(x) = k$ has two solution, then $k \in$
- (D) If $f(x) = k$ has more than two solution, then $k \in$

Column-II

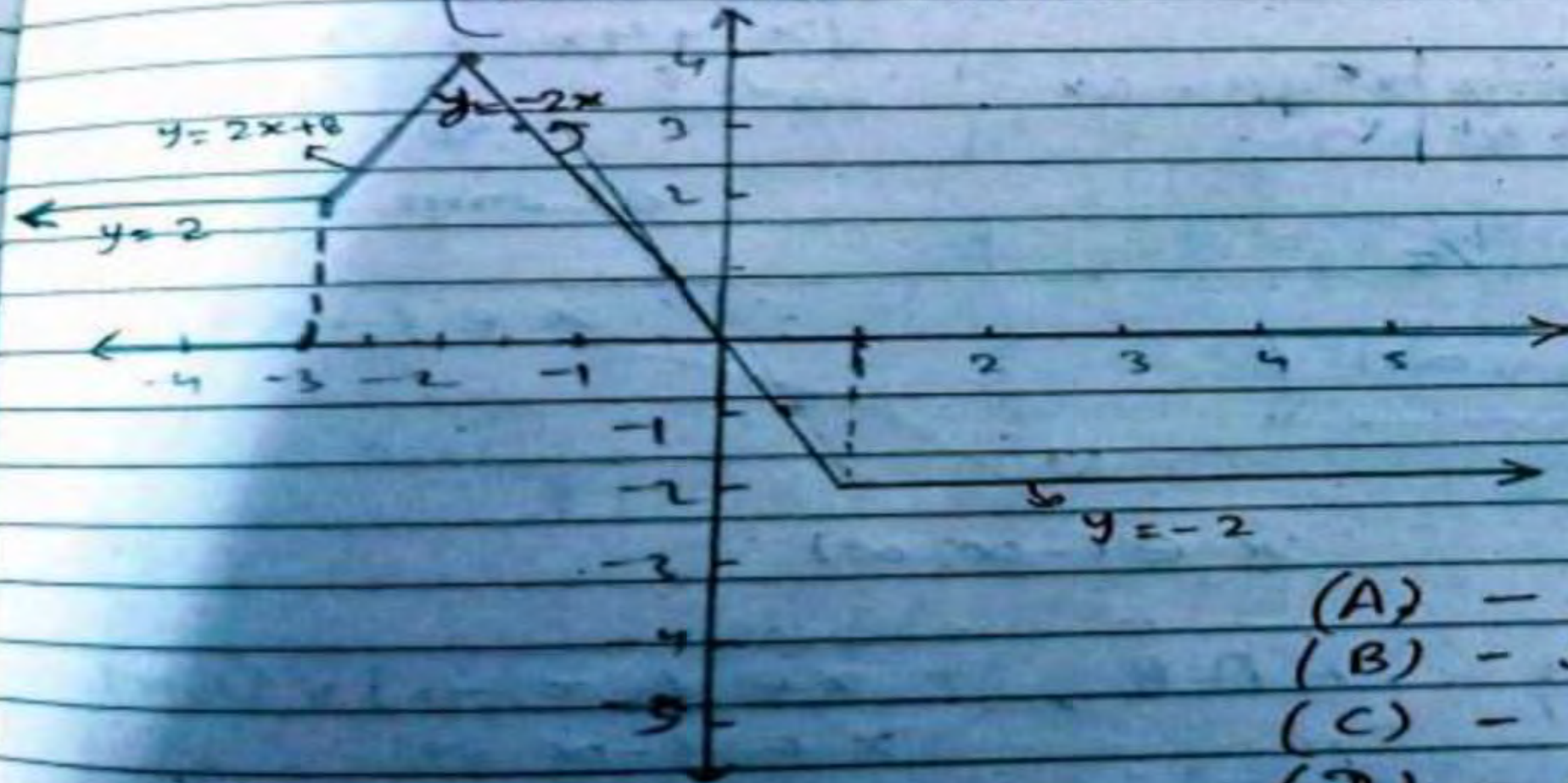
- (p) $(2, 4)$
- (q) $(-\infty, -2) \cup (4, \infty)$
- (r) $(-2, 2) \cup \{4\}$
- (s) $\{-2, 2\}$

[IAH-04]

$$f(x) = |x-1| - 2|x+2| + |x+3|$$

-ve	-ve	-ve	+ve
-ve	-ve	+ve	+ve
-ve	+ve	+ve	+ve

$$y = \begin{cases} -x+1+2x+4-x-3 & x < -3 \\ -x+1+2x+4+x+3 & -3 \leq x \leq -2 \\ -x+1-2x-4+x+3 & -2 \leq x \leq 1 \\ x-1-2x-4+x+3 & x > 1 \end{cases} = \begin{cases} 2 & x < -3 \\ 2x+8 & -3 \leq x \leq -2 \\ -2x & -2 \leq x \leq 1 \\ -2 & x > 1 \end{cases}$$



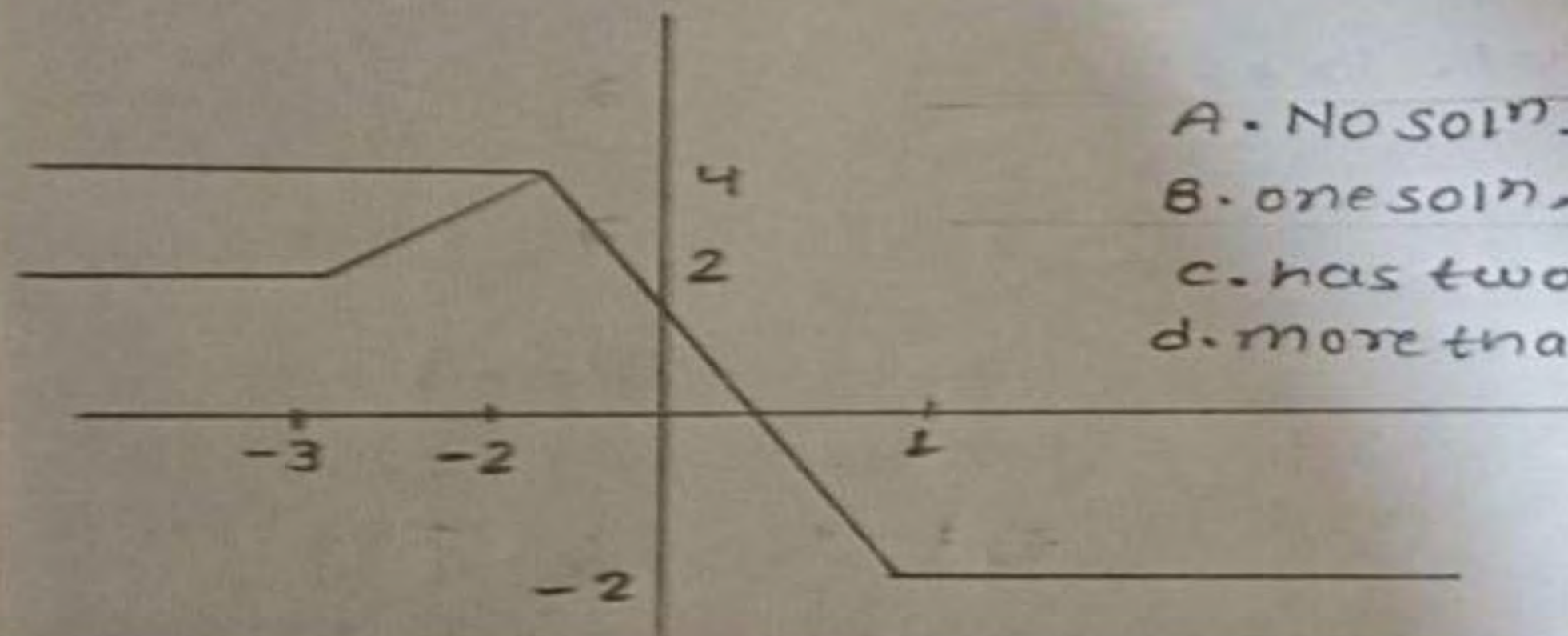
	$-2x$	$-2 \leq x \leq 1$
x	-1.5	
y	3	

- (A) $-9 = (-\infty, -2) \cup (4, \infty)$
 (B) $-9 = (-2, 2) \cup \{4\}$
 (C) $-9 = (2, 4)$
 (D) $-9 = \{-2, 2\}$

TAH 04: $f(x) = |x-2| - 2|x+2| + |x+3|$

T_1	-ve	-ve	-ve	+ve
T_2	-ve	-ve	+ve	+ve
T_3	-ve	+ve	+ve	+ve

$$f(x) = \begin{cases} -x+1+2x+4-x-3 & x \leq -3 \\ -x+1+2x+4+x+3 & -3 < x < -2 \\ -x+1-2x-4+x+3 & -2 \leq x < 1 \\ x-1-2x-4+x+3 & x \geq 1 \end{cases} = \begin{cases} 2 & x \leq -3 \\ 2x+8 & -3 < x < -2 \\ -2x & -2 \leq x < 1 \\ -2 & x \geq 1 \end{cases}$$



- A. No soln $\rightarrow k \in (-\infty, -2) \cup (4, \infty)$
 B. one soln $\rightarrow k \in (-2, 2) \cup \{4\}$
 C. has two soln $\rightarrow k \in (2, 4)$
 D. more than two soln $\rightarrow k \in \{-2, 2\}$

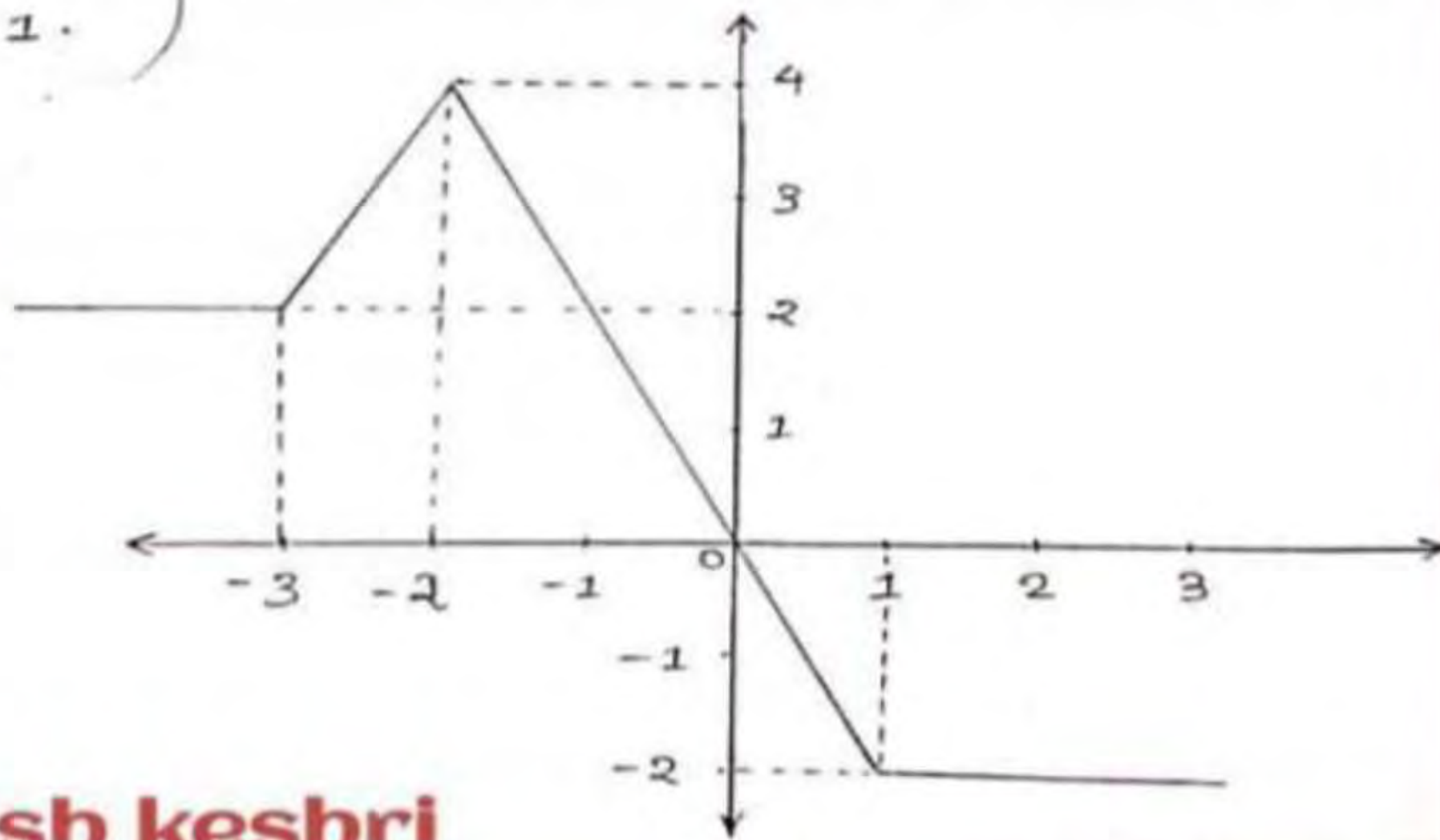
Tah-04.

Consider the funcⁿ $f(x) = |x-1| - 2|x+2| + |x+3|$.

$$\Rightarrow \left. \begin{aligned} & -x+1+2x+4-x-3, \quad x \leq -3. \\ & -x+1+2x+4+x+3, \quad -3 < x \leq -2 \\ & -x+1-2x-4+x+3, \quad -2 < x < 1 \\ & x-1-2x-4+x+3, \quad x \geq 1. \end{aligned} \right\}$$

$$\Rightarrow \left. \begin{aligned} & 2, \quad x \leq -3 \\ & 2x+8, \quad -3 < x \leq -2 \\ & -2x, \quad -2 < x < 1 \\ & -2, \quad x \geq 1 \end{aligned} \right\}$$

$$\begin{array}{l} T_1 \quad -ve \quad -ve \quad -ve \quad +ve \\ T_2 \quad -ve \quad -ve \quad +ve \quad +ve \\ T_3 \quad -ve \quad +ve \quad +ve \quad +ve \end{array}$$



$$(A) \rightarrow K \in (-\infty, -2) \cup (4, \infty).$$

$$(B) \rightarrow K \in (-2, 2) \cup \{4\}.$$

$$(C) \rightarrow K \in (2, 4).$$

$$(D) \rightarrow K \in \{-2, 2\}.$$

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QUESTION



Solution of the inequality, $x - 3 < \sqrt{x^2 + 4x - 5}$

- A** $(-\infty, -5] \cup [1, \infty)$
- B** $(-\infty, -5] \cup (7/5, \infty)$
- C** $(-5, 3]$
- D** $(7/5, \infty)$

Taksh

$$x-3 < \sqrt{x^2+4x-5}$$

$$\text{also } x^2+4x-5 \geq 0$$

Sakshi

(C1) if $x-3 \geq 0$
 $x \geq 3$

$$(x+5)(x-1) \geq 0$$

$$x \in (-\infty, -5] \cup [1, \infty) \rightarrow \textcircled{A}$$

PAGE NO.

DATE:

non-ve

$$x-3 < \sqrt{x^2+4x-5} \text{ --- non-ve}$$

$$x^2+9-6x-x^2-4x+5 < 0$$

$$14-10x < 0$$

$$x > 7/5$$

$$x > 3$$

(C2)

$$x-3 < 0 \Rightarrow x < 3$$

$$x-3 < \sqrt{x^2+4x-5}$$

-ve

non-ve

always true

union

$$x < 3$$

$$x \in (-\infty, \infty) \rightarrow \textcircled{B}$$

Final ans : $A \cap B \Rightarrow (-\infty, -5] \cup [1, \infty)$ Ans (A)

10h-05



Simplify: $x-3 < \sqrt{x^2+4x-5}$.

Case ① if $x-3 \geq 0 \Rightarrow x \geq 3$.

$$\Rightarrow x-3 < \sqrt{x^2+4x-5}$$

SBS.

$$\Rightarrow x^2-6x+9 < x^2+4x-5$$

$$\Rightarrow 10x > 14 \Rightarrow x > \frac{14}{10} \Rightarrow x > \frac{7}{5} \quad \left. \begin{array}{l} \text{---} \\ \text{---} \end{array} \right\} x \in [3, \infty).$$

$$\Rightarrow x^2+4x-5 \geq 0$$

$$\Rightarrow (x-1)(x+5) \geq 0$$

$$\Rightarrow x \in (-\infty, -5] \cup [1, \infty).$$

Ⓐ

Case ② if $x-3 < 0 \Rightarrow x < 3$.

$$\Rightarrow x-3 < \sqrt{x^2+4x-5}$$

Always true $\Rightarrow x \in \mathbb{R}$.

$$x \in (-\infty, 3)$$

$$\cup \Rightarrow x \in (-\infty, \infty)$$

Ⓑ

$$\Rightarrow A \cap B \Rightarrow x \in (-\infty, -5] \cup [1, \infty) \quad \underline{\text{Ans.}}$$

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1 AH-05

$$x-3 < \sqrt{x^2+4x-5}$$

$$\sqrt{x^2+4x-5} > x-3$$

$$x^2+4x-5 \geq 0$$

$$(x+5)(x-1) \geq 0$$

$$x \in (-\infty, -5] \cup [1, \infty)$$

Case (i) $x^2+4x-3 > 0$
 $x > 3$

$$\sqrt{x^2+4x-5} > x-3$$

non-ve

S.B.S

$$x^2+4x-5 > x^2+9-6x$$

$$10x-14 > 0$$

$$x > 7/5$$

$$x \in (7/5, \infty)$$

U

$$x \in (-\infty, \infty)$$

Case (ii) $x-3 < 0$
 $x < 3$

$$\sqrt{x^2+4x-5} > x-3$$

non-ve

↓

always true

$$\therefore x \in (-\infty, 3)$$

Final Ans: $A \cap B$, $x \in (-\infty, -5] \cup [1, \infty)$
 $x \in (-\infty, \infty)$

$$x \in (-\infty, -5] \cup [1, \infty)$$

A

Solve following inequalities

$$(i) \quad \frac{\sqrt{2x-1}}{x-2} < 1$$

$$\text{Ans. } \left[\frac{1}{2}, 2\right) \cup (5, \infty)$$

$$(ii) \quad x - \sqrt{1 - |x|} < 0$$

$$\text{Ans. } [-1, (\sqrt{5} - 1)/2)$$

$$(iii) \quad \sqrt{x^2 - x - 6} < 2x - 3$$

$$\text{Ans. } x \in [3, \infty)$$

$$(iv) \quad \sqrt{x^2 - 6x + 8} \leq \sqrt{x + 1}$$

$$\text{Ans. } x \in \left[\frac{7-\sqrt{21}}{2}, 2\right] \cup \left[4, \frac{7+\sqrt{21}}{2}\right]$$

Tah-06.

$$(i) \frac{\sqrt{2x-1}}{x-2} < 1$$

$$\Rightarrow 2x-1 \geq 0, x-2 \neq 0$$

$$\Rightarrow x \geq \frac{1}{2}, x \neq 2 \quad (A)$$

Case ① if $x-2 > 0$

$$\Rightarrow \sqrt{2x-1} < x-2 \quad (SBS)$$

$$\Rightarrow 2x-1 < x^2-4x+4$$

$$\Rightarrow x^2-6x+5 > 0$$

$$\Rightarrow (x-5)(x-1) > 0$$

$$\Rightarrow x \in (-\infty, 1) \cup (5, \infty) \quad x \in (5, \infty)$$

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Case-② if $x-2 < 0$

$$\Rightarrow \frac{\sqrt{2x-1}}{\underbrace{(x-2)}_{-ve}} < 1$$

$$\Rightarrow \underbrace{\text{any } -ve < 1}_{\text{always true}}$$

$$\Rightarrow x \in \mathbb{R}$$

$$x \in (-\infty, 2)$$

$$\cup \quad x \in (-\infty, 2) \cup (5, \infty) \quad (B)$$

$$\Rightarrow A \cap B$$

$$\Rightarrow x \in \left[\frac{1}{2}, 2\right) \cup (5, \infty) \quad \underline{\text{Ans}}$$

TAH 06: solve following inequalities.

(i) $\frac{\sqrt{2x-1}}{x-2} < 1$

$\sqrt{2x-1} > x-2$; $2x-1 \geq 0 \Rightarrow x \geq \frac{1}{2}$ (A)

Case 1 If $x-2 \geq 0$
 $x \geq 2$

$2x-1 < (x-2)^2$

$2x-1 < x^2+4-4x$

$x^2-6x+5 > 0$

$x^2-5x-x+5 > 0$

$(x(x-5)-1(x-5)) > 0$

$(x-1)(x-5) > 0$

$x \in (-\infty, 1) \cup (5, \infty)$

~~$x \in (-\infty, 1) \cup (5, \infty)$~~

~~$x \in (-\infty, 1) \cup (5, \infty)$~~

$x \in (5, \infty)$

Case 2 If $x-2 < 0$
 $x < 2$

$\sqrt{2x-1} > x-2$

↓
Always true

$x \in (-\infty, 2)$

$x \in (-\infty, 2) \cup (5, \infty)$ (B)

$x \in [\frac{1}{2}, 2) \cup (5, \infty)$

Prob (2) (ii) $x - \sqrt{1 - |x|} < 0$.

$$\Rightarrow x < \sqrt{1 - |x|}$$

$$\Rightarrow 1 - |x| \geq 0$$

$$\Rightarrow |x| \leq 1$$

$$\Rightarrow x \in [-1, 1]. \quad \textcircled{A}$$

Case ① if $x \geq 0$.

$$\Rightarrow x < \sqrt{1 - |x|} \quad (\text{OBS})$$

$$\Rightarrow x^2 < 1 - x$$

$$\Rightarrow x^2 + x - 1 < 0$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{5}}{2}$$

$$x \in \left[0, \frac{-1 + \sqrt{5}}{2}\right)$$

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Case ② if $x < 0$.

$$\Rightarrow x < \sqrt{1 - |x|}$$

$$\Rightarrow x < \sqrt{1 + x}$$

$\downarrow$ $\downarrow$ Non
 -ve -ve

$$\Rightarrow (\text{Always true})$$

$x \in \mathbb{R}$

$$x \in (-\infty, 0)$$

$$\cup x \in \left(-\infty, \frac{\sqrt{5} - 1}{2}\right)$$

$\textcircled{B}$

$$\Rightarrow A \cap B$$

$$\Rightarrow x \in \left[-1, \frac{\sqrt{5} - 1}{2}\right) \quad \underline{\text{Ans.}}$$

Let $S = \{x \in \mathbb{R} : (\sqrt{3} + \sqrt{2})^x + (\sqrt{3} - \sqrt{2})^x = 10\}$. Then the number of elements in S is :

- A** 4
- B** 0
- C** 2
- D** 1

TAM-07

$$S = \{x \in \mathbb{R} : (\sqrt{3} + \sqrt{2})^x + (\sqrt{3} - \sqrt{2})^x = 10\}$$

$$(\sqrt{3} + \sqrt{2})(\sqrt{3} - \sqrt{2}) = 3 - 2 = 1$$

$$\sqrt{3} - \sqrt{2} = \frac{1}{\sqrt{3} + \sqrt{2}}$$

Now,

$$(\sqrt{3} + \sqrt{2})^x + \frac{1}{(\sqrt{3} + \sqrt{2})^x} = 10$$

$$[(\sqrt{3} + \sqrt{2})^x = t]$$

$$t + \frac{1}{t} = 10$$

$$t^2 - 10t + 1 = 0$$

$$t = \frac{10 \pm \sqrt{100 - 4}}{2} = \frac{10 \pm 4\sqrt{6}}{2} = 5 \pm 2\sqrt{6}$$

$$t = 5 \pm 2\sqrt{6} = (\sqrt{3} \pm \sqrt{2})^2$$

$$(\sqrt{3} + \sqrt{2})^x = (\sqrt{3} + \sqrt{2})^2 \quad \text{or} \quad (\sqrt{3} - \sqrt{2})^2$$

$$[x = 2]$$

$$(\sqrt{3} + \sqrt{2})^x = \left(\frac{1}{\sqrt{3} + \sqrt{2}}\right)^2$$

$$(\sqrt{3} + \sqrt{2})^x = (\sqrt{3} + \sqrt{2})^{-2}$$

$$[x = -2]$$

no. of element in S is 2.

$$S = \{2, -2\}$$

Tah-07. (Thala for a reason).

Let $S = \{x \in \mathbb{R} : (\sqrt{3} + \sqrt{2})^x + (\sqrt{3} - \sqrt{2})^x = 10\}$. Then the no. of element in S is:

$$\Rightarrow (\sqrt{3} + \sqrt{2})^x + (\sqrt{3} - \sqrt{2})^x = 10.$$

$$\Rightarrow t + \frac{1}{t} = 10$$

$$\Rightarrow t^2 + 1 - 10t = 0$$

$$\Rightarrow t^2 - 10t + 1 = 0$$

$$\Rightarrow t = \frac{10 \pm \sqrt{96}}{2} \Rightarrow t = \frac{5 \pm 2\sqrt{6}}{1}$$

$$\text{let: } (\sqrt{3} + \sqrt{2})^x = t.$$

And:

$$\Rightarrow \frac{1}{\sqrt{3} + \sqrt{2}} \times \frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} - \sqrt{2}}$$

$$\Rightarrow \frac{\sqrt{3} - \sqrt{2}}{1}$$

So;

$$\Rightarrow (\sqrt{3} - \sqrt{2})^x = \frac{1}{t}$$

$$\# (\sqrt{3} + \sqrt{2})^x = 5 + 2\sqrt{6}$$

$$(\sqrt{3} + \sqrt{2})^x = (\sqrt{3})^2 + 2 \cdot \sqrt{3} \cdot \sqrt{2} + (\sqrt{2})^2$$

$$(\sqrt{3} + \sqrt{2})^x = (\sqrt{3} + \sqrt{2})^2$$

$$x = 2$$

$$\# (\sqrt{3} + \sqrt{2})^x = 5 - 2\sqrt{6}$$

$$\Rightarrow (\sqrt{3} + \sqrt{2})^x = (\sqrt{3} - \sqrt{2})^x$$

$$\Rightarrow (\sqrt{3} + \sqrt{2})^x = (\sqrt{3} + \sqrt{2})^{-2}$$

$$x = -2$$

$$\#(S) = 2 \text{ Ans.}$$

The number of real roots of the equation $\sqrt{x^2 - 4x + 3} + \sqrt{x^2 - 9} = \sqrt{4x^2 - 14x + 6}$, is

- A** 0
- B** 1
- C** 3
- D** 2

Tah-08



$$\sqrt{x^2-4x+3} + \sqrt{x^2-9} = \sqrt{4x^2-14x+6} \text{ is :}$$

$$\Rightarrow \sqrt{(x-3)(x-1)} + \sqrt{(x-3)(x+3)} = \sqrt{(x-3)(4x-2)} .$$

$$\Rightarrow \sqrt{(x-3)(x-1)} + \sqrt{(x-3)(x+3)} - \sqrt{(x-3)(4x-2)} = 0 .$$

$$\Rightarrow \underbrace{\sqrt{(x-3)}}_{=0} \left\{ \underbrace{\sqrt{(x-1)} + \sqrt{(x+3)} - \sqrt{(4x-2)}}_{=0} \right\} = 0 .$$

$$\begin{array}{l} \star \quad \sqrt{x-3} = 0 \\ \Rightarrow x-3 = 0 \\ \Rightarrow \boxed{x=3} \end{array} \quad \left| \quad \begin{array}{l} \Rightarrow \sqrt{x-1} + \sqrt{x+3} = \sqrt{4x-2} \quad (\text{S.B.S}) \\ \Rightarrow x-1 + x+3 + 2 \cdot \sqrt{x-1} \cdot \sqrt{x+3} = 4x-2 . \\ \Rightarrow 2x+2 + 2\sqrt{x^2+2x-3} = 4x-2 . \\ \Rightarrow 2\sqrt{x^2+2x-3} = 2x-4 \\ \Rightarrow \sqrt{x^2+2x-3} = x-2 \quad (\text{S.B.S}) \\ \Rightarrow x^2+2x-3 = x^2-4x+4 \\ \Rightarrow 6x=7 \Rightarrow \boxed{x=\frac{7}{6}} \times \end{array} \right.$$

krish keshri
jharkhand

$\Rightarrow$ no of roots = 1 Ans.

because doesn't satisfy $\sqrt{x^2-9}$.

Q-5: The number of real roots of the equation:

$$\sqrt{x^2 - 4x + 3} + \sqrt{x^2 - 9} = \sqrt{4x^2 - 14x + 6} \text{ is!}$$

- Ⓐ 0. Ⓑ 1 Ⓒ 3 Ⓓ 2.

TAH 08
BY REED
FROM WB

Soln: $\sqrt{x^2 - 4x + 3} + \sqrt{x^2 - 9} = \sqrt{4x^2 - 14x + 6}$

or, $\sqrt{(x-1)(x-3)} + \sqrt{(x+3)(x-3)} = \sqrt{4x^2 - 12x - 2x + 6}$

or, $\sqrt{(x-1)(x-3)} + \sqrt{(x+3)(x-3)} = \sqrt{(4x-2)(x-2)} \quad \text{--- (A)}$

or, $\sqrt{x-3} [\sqrt{x-1} + \sqrt{x+3}] = \sqrt{4x-2} \cdot \sqrt{x-2}$

or, $\sqrt{x-3} [\sqrt{x-1} + \sqrt{x+3}] - \sqrt{x-3} \sqrt{4x-2} = 0$

or, $\sqrt{x-3} [\sqrt{x-1} + \sqrt{x+3} - \sqrt{4x-2}] = 0$

$\therefore \sqrt{x-3} = 0$ or, $\sqrt{x-1} + \sqrt{x+3} - \sqrt{4x-2} = 0$

or, $x-3 = 0$

or, $x = 3$

(valid) $\therefore$ no problem in Ⓐ.

$\therefore x = 3 \rightarrow$ only soln

$\therefore$ Ans $\rightarrow$ No. of real roots of the eqn is $\Rightarrow$ Ⓑ 1.

or, $\sqrt{x-1} + \sqrt{x+3} = \sqrt{4x-2}$

S.B.S.

or, $x-1 + x+3 + 2\sqrt{x-1} \cdot \sqrt{x+3} = 4x-2$

or, $2x+2 + 2\sqrt{x^2+2x-3} = 4x-2$

or, $2\sqrt{x^2+2x-3} = 2x-4 = 2(x-2)$

or, $\sqrt{x^2+2x-3} = x-2$

S.B.S.

or, $x^2+2x-3 = x^2 - 4x + 4$

or, $6x = 7$

or, $x = \frac{7}{6}$

$\left\{ \frac{7}{6} \rightarrow < 3 \right\}$

Discarded

bcz $x^2 - 9 \Rightarrow$ -ve @ $x = \frac{7}{6}$
i.e. $\sqrt{x^2 - 9}$ is NOT DEFINED.

QUESTION



Solve: $\sqrt{x^2 - 6x + 9} + \sqrt{x^2 + 8x + 16} = 7$

[TAH-09]

$$\sqrt{x^2 - 6x + 9} + \sqrt{x^2 + 8x + 16} = 7$$

$$\sqrt{(x-3)^2} + \sqrt{(x+4)^2} = 7$$

$$|x-3| + |x+4| = 7$$

$\hookrightarrow a$

$\hookrightarrow b$

$$|a| + |b| = |a-b| \iff ab \leq 0$$

Now

$$(x-3)(x+4) \leq 0$$



$$x \in [-4, 3]$$

Tah-09



Solve : $\sqrt{x^2 - 6x + 9} + \sqrt{x^2 + 8x + 16} = 7$.

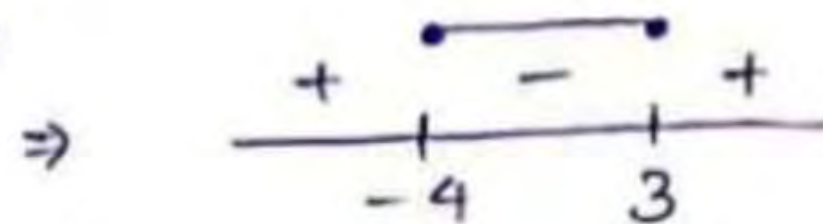
$$\Rightarrow \sqrt{(x-3)(x-3)} + \sqrt{(x+4)(x+4)} = 7$$

$$\Rightarrow \sqrt{(x-3)^2} + \sqrt{(x+4)^2} = 7$$

$$\Rightarrow |x-3| + |x+4| = 7 \rightarrow |x-3 - x-4| = |-7| = \textcircled{7}$$

$$\Rightarrow |a| + |b| = |a-b|$$

$$\Rightarrow (x-3)(x+4) \leq 0$$



$$\Rightarrow x \in [-4, 3] \text{ Ans.}$$

krish keshri
jharkhand

The sum of the solutions of the equation $|\sqrt{x} - 2| + \sqrt{x}(\sqrt{x} - 4) + 2 = 0$ ($x > 0$) is equal to:

- A** 9
- B** 12
- C** 4
- D** 10

Tah-10.

Sum of the solⁿ of the eqⁿ :

$$|\sqrt{x}-2| + \sqrt{x}(\sqrt{x}-4) + 2 = 0 \quad (x > 0) \text{ is equal to :}$$

$$\Rightarrow \underline{\text{Case-(1)}} \quad \sqrt{x}-2 \geq 0$$

$$\Rightarrow \sqrt{x} \geq 2 \Rightarrow x \geq 4.$$

$$\Rightarrow \sqrt{x}-2 + x - 4\sqrt{x} + 2 = 0$$

$$\Rightarrow x - 3\sqrt{x} = 0$$

$$\Rightarrow \sqrt{x}(\sqrt{x}-3) = 0$$

$$\Rightarrow \begin{array}{l|l} \sqrt{x} = 0 & \sqrt{x} - 3 = 0 \\ x = 0 & x = 9 \\ \times & \checkmark \end{array}$$

$$\Rightarrow x = 1, 9$$

$$\Rightarrow \text{Sum} = 10 \quad \underline{\text{Ans.}}$$

$$\Rightarrow \underline{\text{Case-(2)}} \quad \sqrt{x}-2 < 0.$$

$$\Rightarrow x < 4.$$

$$\Rightarrow -\sqrt{x} + 2 + x - 4\sqrt{x} + 2 = 0.$$

$$\Rightarrow x - 5\sqrt{x} + 4 = 0.$$

$$\Rightarrow (\sqrt{x}-1)(\sqrt{x}-4) = 0$$

$$\Rightarrow \begin{array}{l|l} \sqrt{x} = 1 & \sqrt{x} = 4 \\ x = 1 \checkmark & x = 16 \times \end{array}$$

krish keshri
jharkhand

Q-8! Sum of solutions of $|\sqrt{x}-2| + \sqrt{x}(\sqrt{x}-4) + 2 = 0$, $(x > 0)$

Soln $|\sqrt{x}-2| + \sqrt{x}(\sqrt{x}-4) + 2 = 0$.

**TAH 10
BY REED
FROM WB**

Case-1! $(\sqrt{x}-2) \geq 0 \Rightarrow x \geq 4$.

$\therefore \sqrt{x}-2 + x - 4\sqrt{x} + 2 = 0$

$\Rightarrow x - 3\sqrt{x} = 0$

$\Rightarrow \sqrt{x}(\sqrt{x}-3) = 0$

$\Rightarrow x = 0$ $\sqrt{x} = 3$
 $\Rightarrow x = 9$
Discarded

$x = 9$

Case-2! $\sqrt{x}-2 < 0 \Rightarrow x < 4$

$\therefore 2 - \sqrt{x} + x - 4\sqrt{x} + 2 = 0$

$\Rightarrow x - 5\sqrt{x} + 4 = 0$

$\Rightarrow x - 4\sqrt{x} - \sqrt{x} + 4 = 0$

$\Rightarrow (\sqrt{x}-1)(\sqrt{x}-4) = 0$

$\downarrow$

$x = 1$

OR

$x = 16$

accepted

Rejected

$x = 1$

$x = 1$

$\therefore x = 1, 9$

$\therefore$ Sum of roots $= 1 + 9 = 10$ (Ans = (d))

Solution to Previous BPPs

Bumper Practice Problems



$$1. \sqrt[3]{\frac{x-2}{x-1}} < \sqrt[3]{\frac{1}{x-1}}$$

$$2. \sqrt{3x-2} < \sqrt{x+4}$$

$$3. \sqrt{4x-3} < \sqrt{2x+5}$$

$$4. \sqrt[3]{(3x-5)} < (x-1)$$

$$5. \sqrt[3]{3x-2} < x$$

$$6. \sqrt{(x+14)} < (x+2)$$

$$7. \sqrt{2x-2} < x-1$$

$$8. \sqrt{-x^2+4x-3} > (6-2x)$$

$$9. \sqrt{x-2} + \sqrt{x-1} > 2$$

$$10. 7\sqrt{x} + 8\sqrt{-x} + \frac{15}{x^3} = 98$$

$$11. \sqrt{2x-4} - \sqrt{x+5} = 1$$

$$12. \sqrt{x-1} + \sqrt{2x+6} = 6$$

1. $x \in (1, 3)$

2. $x \in \left[\frac{2}{3}, 6\right)$

3. $x \in \left[\frac{3}{4}, 1\right)$

4. $x \in (-1, \infty) - \{2\}$

5. $x \in (-2, 1) \cup (1, \infty)$

6. $x \in (2, \infty)$

7. $x \in (3, \infty)$

8. $x \in \left(\frac{13}{5}, 3\right)$

9. $x \in \left(\frac{41}{16}, \infty\right)$

BPP

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1. $\sqrt[3]{\frac{x-2}{x-1}} < \sqrt[3]{\frac{1}{x-1}}$

$\Rightarrow$ on cubing both sides $\rightarrow$

$$\Rightarrow \left(\frac{x-2}{x-1} \right)^{\frac{1}{3} \times 3} < \left(\frac{1}{x-1} \right)^{\frac{1}{3} \times 3}$$

$$\Rightarrow \frac{x-2}{(x-1)} < \frac{1}{(x-1)}$$

$$\Rightarrow \frac{x-2}{x-1} - \frac{1}{(x-1)} < 0$$

$$\Rightarrow \frac{(x-2) - (1)}{(x-1)} < 0$$

$$\Rightarrow \frac{x-3}{x-1} < 0$$

$$\Rightarrow x \in \underline{(1, 3)}$$

$$2. \sqrt{3x-2} < \sqrt{x+4}$$

on Sq. both sides,

$$3x-2 \geq 0$$

$$x \geq \frac{2}{3}$$

$$x+4 \geq 0$$

$$x \geq -4$$

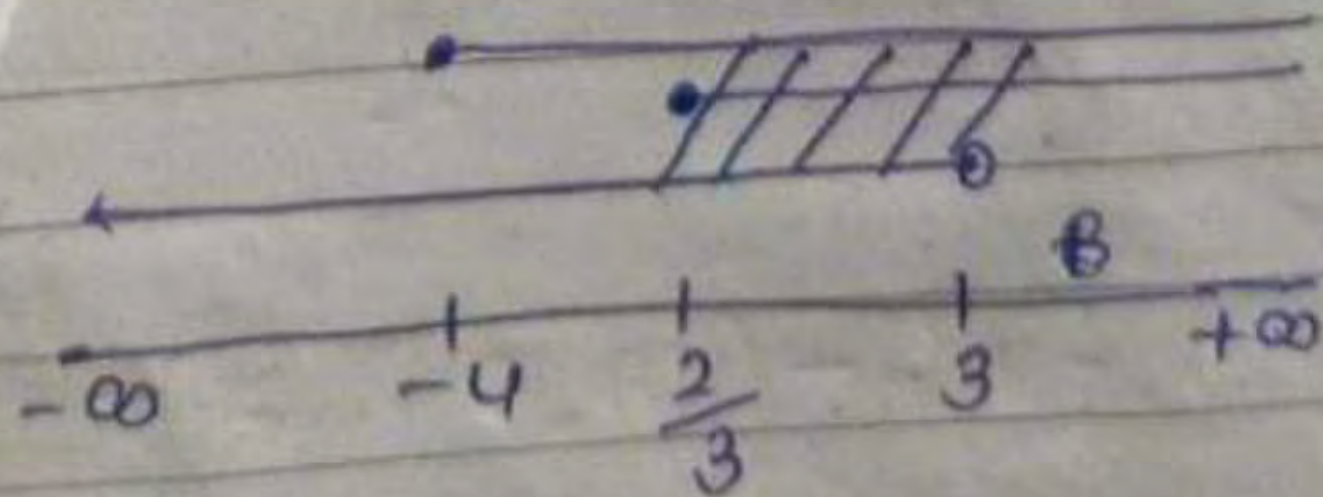
$$2x-6 < 0$$

$$x < 3$$

$$3x-2 < x+4$$

$$2x-2-4 < 0$$

$$x < 3$$



$$x \in \left[\frac{2}{3}, 3 \right)$$

3. $\sqrt{4x-3} < \sqrt{2x+5}$

$\sqrt{4x-3} < \sqrt{2x+5}$

SBS

$4x-3 < 2x+5$

$2x < 8$

$x < 4$

$4x-3 \geq 0$

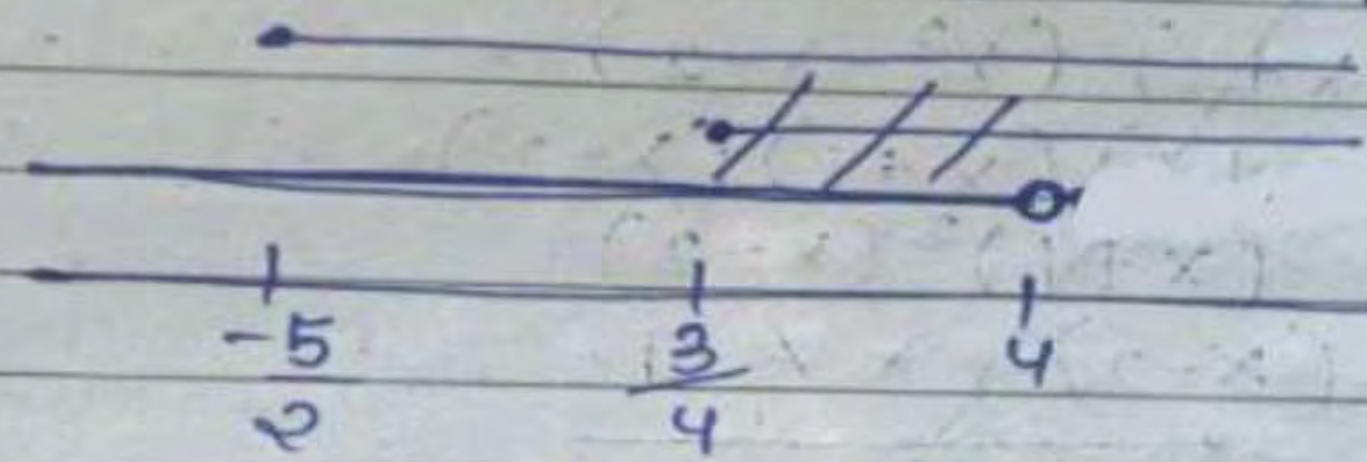
$4x \geq 3$

$x \geq \frac{3}{4}$

$2x+5 \geq 0$

$2x \geq -5$

$x \geq -\frac{5}{2}$



$x \in \left[\frac{3}{4}, 4 \right)$

4.

$$\sqrt[3]{3x-5} < (x-1)$$

cubing both sides.

$$3x-5 < (x-1)^3$$

$$3x-5 < x^3 - 1^3 - 3x(x-1)$$

$$0 < x^3 - 3x^2 + 3x - 1 - 3x + 5$$

$$0 < x^3 - 3x^2 + 4$$

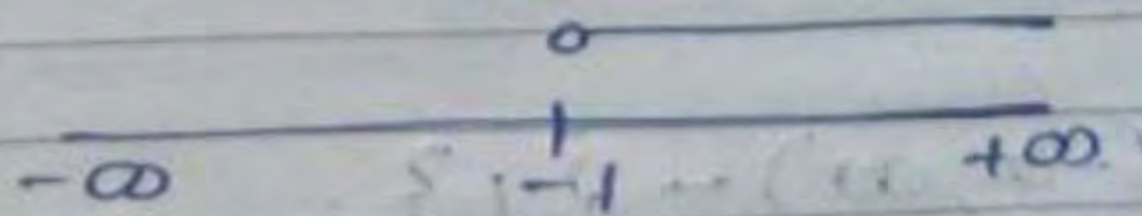
$$0 < x^2(x+1) - 4x(x+1) + 4(x+1)$$

$$0 < (x+1)(x^2 - 4x + 4)$$

$$0 < (x+1)(x-2)^2$$

$$(x+1)(x-2)^2 > 0$$

$$(x+1) > 0, \quad x \neq 2$$



$$x \in (-1, \infty) - \{2\}$$

5:

$$\sqrt[3]{3x-2} < x$$

cubing $\rightarrow$

$$3x-2 < x^3$$

$$0 < x^3 - 3x + 2$$

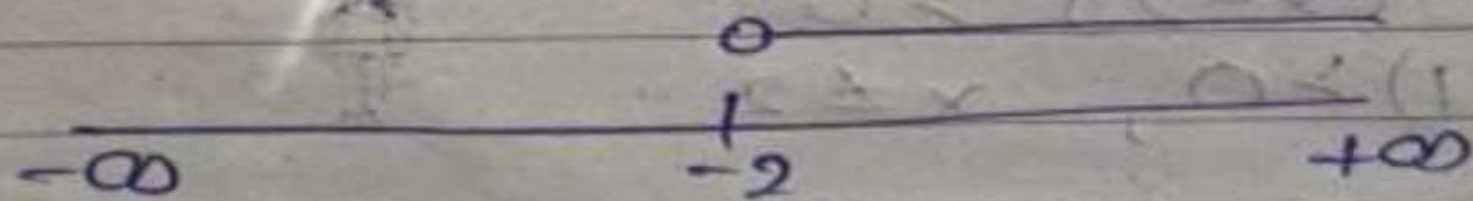
$$x^3 - 3x + 2 > 0$$

$$x^2(x-1) + x(x-1) - 2(x-1) > 0$$

$$(x-1)(x^2+x-2) > 0$$

$$(x-1)(x-1)(x+2) > 0$$

$$(x+2) > 0, x \neq 1$$



$$x \in (-2, \infty) - \{1\}$$

BPP-6

$$\sqrt{n+14} < (n+2)$$

case (i)

$$n+1$$

$$n+2 > 0 \Rightarrow n > -2$$

case (ii)

$$n+2 < 0$$

not possible

By S.B.S., $n+14 < n^2 + 4n + 4$

$$n^2 + 3n - 10 > 0$$

$$(n+5)(n-2) > 0$$

$$\therefore n \in (2, \infty)$$

On taking union of case (i) & (ii).

$$n \in (2, \infty) \text{ --- (A)}$$

$$n+14 > 0$$

$$\Rightarrow n > -14$$

$$n \in [-14, \infty) \text{ --- (B)}$$

$$\therefore A \cap B \Rightarrow n \in (2, \infty)$$

$$6) \sqrt{x+14} < (x+2)$$

Case-01

$$x+2 \geq 0 \rightarrow x \geq -2$$

$\sqrt{x+14} < (x+2)$ is always +ve
always +ve.

SBS

$$\therefore x+14 < x^2+4+4x$$

$$x^2+3x-10 > 0$$

$$(x+5)(x-2) > 0$$

$$x \in (-\infty, -5) \cup (2, \infty)$$

$$\therefore x \in (2, \infty)$$

$$\therefore x \in (2, \infty)$$

Date _____
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Case-02

$$x+2 < 0$$

$$x < -2$$

$$\therefore \sqrt{x+14} < (x+2)$$

Case +ve $\downarrow$ -ve

Not possible

$$x \in \emptyset$$

$$x+14 \geq 0$$

$$\therefore x \geq -14$$

$$7. \sqrt{2x-2} < x-1$$

Case - (1)

$$x-1 \geq 0$$

$$x \geq 1 \Rightarrow x \in [1, \infty)$$

SBS

$$2x-2 < x^2-2x+1$$

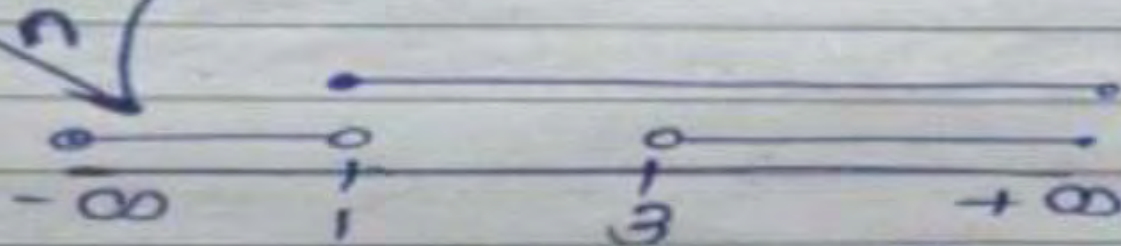
$$0 < x^2-4x+3$$

$$0 < x^2-3x-x+3$$

$$0 < x(x-3)-1(x-3)$$

$$(x-3)(x-1) > 0$$

$$x \in (-\infty, 1) \cup (3, \infty)$$



$$x \in (3, \infty) \quad (B)$$

$$A \cap B =$$

$$x \in (3, \infty)$$

$$\begin{aligned} 2x-2 &\geq 0 \\ 2x &\geq 2 \end{aligned}$$

$$x \geq 1, x \in [1, \infty)$$

(A)

Case - (2)

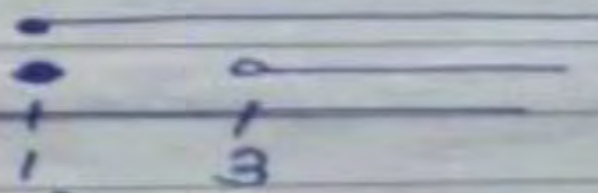
$$x-1 < 0$$

$$x < 1$$

$$\sqrt{2x-2} < x-1$$

Not possible

$$x \in \phi$$



$$7) \sqrt{2x-2} < x-1$$

case-01

$$x+7/0 \therefore x \geq 1$$

$$\sqrt{2x-2} < x-1 \quad \begin{matrix} \swarrow \\ +ve \end{matrix} \quad \begin{matrix} \searrow \\ +ve \end{matrix}$$

$$\therefore 2x-2 < x^2+1-2x$$

$$x^2-4x+3 > 0$$

$$(x+3)(x-1) > 0$$

$$\therefore x \in (-\infty, -3) \cup (1, \infty)$$

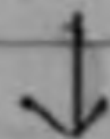
$$\boxed{\therefore x \in (3, \infty)}$$

case-02

$$x-1 < 0$$

$$x < 1$$

$$\sqrt{2x-2} < x-1$$



$$x \in \phi$$

$$2x-2 \geq 0 \therefore x \geq 1$$

$$x \in (3, \infty)$$

$$8) \sqrt{-x^2+4x-3} > (6-2x)$$

Case 1

$$6-2x \geq 0$$

$$\therefore 2x \leq 6$$

$$[x \leq 3]$$

$\therefore$ SBS

$$-x^2+4x-3 > 36+4x^2-24x$$

$$5x^2-28x+39 < 0$$

$$5x^2-15x-13x+39 < 0$$

$$5x(x-3)-13(x-3) < 0$$

$$(5x-13)(x-3) < 0$$

$$[\therefore x \in (13/5, 3)]$$

$$-x^2+4x-3 \geq 0$$

$$x^2-4x+3 \leq 0$$

$$(x-3)(x-1) \leq 0$$

$$\therefore x \in (1, 3)$$

$$\therefore x \in (13/5, 3)$$

Case 2

$$6-2x < 0$$

$$2x > 6$$

$$x > 3$$

$$\therefore \sqrt{x^2+4x-3} > 6-2x$$

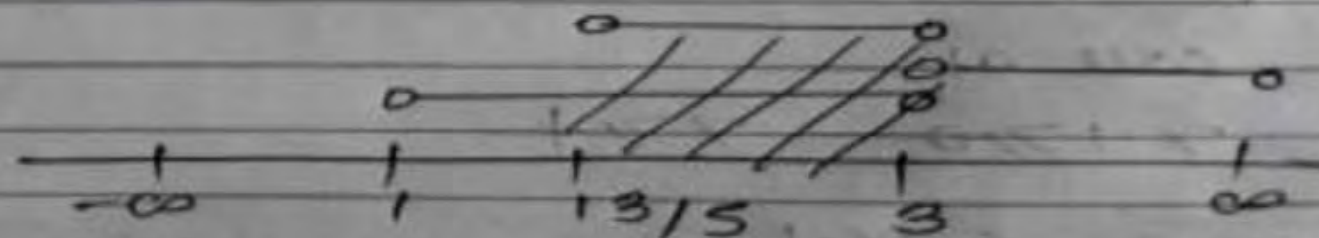
$$\downarrow \quad \downarrow$$

$$+ve \quad -ve$$

always True

$$[\therefore x \in (3, \infty)]$$

$$x \in (13/5, 3) \cup (3, \infty)$$



BPP - 11

S.B.S

$$\sqrt{2n-4} - \sqrt{n+5} = 1$$

$$2n-4+n+5 - 2\sqrt{2(n-2)(n+5)} = 1$$

$$\Rightarrow 3n+1-1 = 2\sqrt{2(n-2)(n+5)}$$

$$\Rightarrow 3n = 2\sqrt{2(n-2)(n+5)}$$

S.B.S

$$\Rightarrow (3n)^2 = 8(n^2+3n-10)$$

$$\Rightarrow 9n^2 - 8n^2 - 24n + 80 = 0$$

$$\Rightarrow n^2 - 24n + 80 = 0$$

$$\Rightarrow (n-20)(n-4) = 0$$

$$\therefore n \in \{4, 20\} \quad \text{--- (A)}$$

$$\therefore A \cap B \Rightarrow n \in \{4, 20\}$$

$$2n-4 > 0$$

$$n > 2$$

$$n+5$$

$$n > -5$$

$$\text{--- (B)}$$

8x10

4x2	x+5	2
2x2	x+5	2

$$11) \sqrt{2x-4} - \sqrt{x+5} = 1$$

SBS

$$2x-4 + (x+5) - 2\sqrt{2x-4} \cdot \sqrt{x+5} = 1$$

$$3x+1 - 2\sqrt{2x-4} \cdot \sqrt{x+5} = 1$$

$$3x = 2\sqrt{2x-4} \cdot \sqrt{x+5}$$

SBS

$$9x^2 = 4[(2x-4)(x+5)]$$

$$9x^2 = 4[2x^2 + 6x - 20]$$

$$9x^2 = 8x^2 + 24x - 80$$

$$x^2 - 24x + 80 = 0$$

$$(x-20)(x-4) = 0$$

$$x = 4, 20$$

$$2x-4 \geq 0 \quad \text{or} \quad x+5 \geq 0$$

$$x \geq 2$$

$$x \geq -5$$

$$\therefore x \in [2, \infty)$$

$$\therefore \text{Ans} \rightarrow x = 4, 20$$

BPP-12

$$\sqrt{n-1} + \sqrt{2n+6} = 6$$

So B.S

$$n-1 + 2n+6 + 2\sqrt{2(n-1)(n+3)} = 36$$

$$\Rightarrow 3n+5 + 2\sqrt{2(n-1)(n+3)} = 36$$

$$\Rightarrow 2\sqrt{2(n-1)(n+3)} = 31-3n$$

Again S.B.S,

$$8(n-1)(n+3) = 961 - 186n + 9n^2$$

$$\Rightarrow 8n^2 + 16n - 24 = 961 - 186n + 9n^2$$

$$\Rightarrow n^2 - 202n + 985 = 0$$

$$\Rightarrow (n-197)(n-5) = 0$$

$$n = 5, 197$$

$$\therefore n \in \{5, 197\} \quad \text{--- (A)}$$

$$n-1 > 0 \quad \& \quad 2n+6 > 0$$

$$n > 1$$

$$n+3 > 0$$

$$n > -3 \quad \text{--- (B)}$$

$$\therefore A \cap B \Rightarrow n \in \{5, 197\}$$

$$12) \sqrt{x-1} + \sqrt{2x+6} = 6$$

SBS

$$x-1 \geq 0$$

$$2x+6 \geq 0$$

$$x \geq 1$$

$$x \geq -3$$

$$(x-1) + (2x+6) + 2\sqrt{x-1}\sqrt{2x+6} = 36$$

$$3x+5 + 2\sqrt{x-1}\sqrt{2x+6} = 36$$

$$\therefore x \in [1, \infty)$$

$$2\sqrt{x-1}\sqrt{2x+6} = 31-3x$$

SBS

$$\therefore 4(x-1)(2x+6) = 961 + 9x^2 - 186x$$

$$4(2x^2+4x-6) = 11$$

$$8x^2+16x-24 = 961+9x^2-186x$$

$$x^2 - 202x + 985 = 0$$

$$x^2 - 197x - 5x + 985 = 0$$

$$x(x-197) - 5(x-197) = 0$$

$$\therefore (x-5)(x-197) = 0$$

$$\therefore x = 197, 5$$



Home Challenge-08



Let $f(x) = e^{\left(\frac{2^{-x}}{1+2^{-x}}\right)}$. If $\prod_{r=-10}^{10} f(\ln 2^r) = e^\lambda$, then find the value of $(2\lambda - 15)$. [Ans. 6]

H.C. - 8: Let $f(x) = e^{\left(\frac{2^{-x}}{1+2^{-x}}\right)}$. If $\prod_{r=-10}^{10} f(\ln 2^r) = e^\lambda$, then find the value of $(2\lambda - 15)$.

Soln:

$$f(x) = e^{\left(\frac{2^{-x}}{1+2^{-x}}\right)} = e^{\left(\frac{1/2^x}{1+1/2^x}\right)} = e^{\left(\frac{1}{2^x+1}\right)}$$

$$f(\ln 2^r) = f(r \ln 2) = e^{\frac{1}{2^{r \ln 2} + 1}}$$

$$E = \prod_{r=-10}^{10} f(r \ln 2) = e^{\frac{1}{1+2^{-10 \ln 2}}} \times e^{\frac{1}{1+2^{-9 \ln 2}}} \times \dots \times e^{\frac{1}{1+2^{0 \ln 2}}} \times \dots \times e^{\frac{1}{1+2^{10 \ln 2}}}$$

$$\Rightarrow E = e^{\frac{1}{1+2^{-10 \ln 2}} + \frac{1}{1+2^{-9 \ln 2}} + \dots + \frac{1}{1+2^{0 \ln 2}} + \dots + \frac{1}{1+2^{10 \ln 2}}}$$

Observe: $\frac{1}{1+2^{\frac{r \ln 2}{1/t}}} + \frac{1}{1+2^{\frac{r \ln 2}{t}}} = \frac{1}{1+\frac{1}{t}} + \frac{1}{1+t}$

$$= \frac{t}{1+t} + \frac{1}{1+t} = 1$$

$$E = e^{(1+1+\dots, 10 \text{ times}) + \frac{1}{1+2^{0 \ln 2}}}$$

$$\Rightarrow E = e^{10 + \frac{1}{1+1}}$$

$$\Rightarrow E = e^{(10 + \frac{1}{2})}$$

$$\Rightarrow E = e^{\frac{21}{2}} = e^\lambda \Rightarrow \lambda = \frac{21}{2}$$

HC 8
BY REED
FROM WB

$$\therefore 2\lambda - 15 = 2 \times \frac{21}{2} - 15 = 21 - 15 = 6$$

Ans.

HC-8

Let $f(x) = e^{\left(\frac{2^{-x}}{1+2^x}\right)}$. if $\prod_{n=1}^{10} f(\ln 2^n) = e^A$, then

Find the value of $(2A-15)$

~~(1) $f(x) = e^{\left(\frac{2^{-x}}{1+2^x}\right)}$~~

$$f(x) = e^{\left(\frac{2^{-x}}{1+2^x}\right)} \rightarrow \left(\frac{2^{-x}}{1+\frac{1}{2^x}}\right) = \frac{1}{1+2^x}$$

$$f(x) = e^{\left(\frac{1}{1+2^x}\right)} \quad (i)$$

$$\prod_{n=1}^{10} e^{\left(\frac{2^{-n \ln 2}}{1+2^{-n \ln 2}}\right)}$$

$$= e^{\sum_{n=1}^{10} \left(\frac{2^{-n \ln 2}}{1+2^{-n \ln 2}}\right)}$$

$$= e^{\left(\frac{2^{-1 \ln 2}}{1+2^{-1 \ln 2}} + \frac{2^{-2 \ln 2}}{1+2^{-2 \ln 2}} + \dots + \frac{2^{-9 \ln 2}}{1+2^{-9 \ln 2}} + \frac{2^{-10 \ln 2}}{1+2^{-10 \ln 2}}\right)}$$

$\downarrow$ by eq (i) $\downarrow$ by eq (i)
 $\frac{1}{1+2^{1 \ln 2}} \quad \quad \frac{1}{1+2^{10 \ln 2}}$

we observed

$$1 = 1^{\text{st}} \text{ term} + 2^{\text{nd}} \text{ term} = 2^{\text{nd}} \text{ term} + 2^{\text{rd}} \text{ term} = \dots = 10^{\text{th}} \text{ term} + 11^{\text{th}} \text{ term}$$

total 10 pairs

$$= e^{\left(10 + \frac{1}{1+2^0}\right)}$$

$$= e^{(10 + \frac{1}{2})} = e^{\frac{21}{2}} = e^A$$

$$A = \frac{21}{2}$$

$$2A - 15 = 21 - 15 = 6$$

THANK
YOU